This is a closed everything exam, except a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so.

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1. (5 Points) Write your personal information below.
   
   Your Name: ____________________________
   
   Your GSI: ____________________________
   
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2. Consider the following map from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \):

\[
T \left( \begin{array}{c} x \\ y \end{array} \right) = \left( \begin{array}{c} x + y - 1 \\ x - y + 1 \end{array} \right).
\]

Is \( T \) a linear transform? Explain.
3. Is the following identity true for all pairs of invertible matrices $A, B \in \mathcal{R}^{n \times n}$?

$$\left((AB)^{-1}\right)^T = \left(A^{-1}\right)^T \left(B^{-1}\right)^T.$$ 

Explain your answer.
4. Give an explicit formula for the components of the vector

\[
\begin{pmatrix} -3 & 4 \\ 4 & 3 \end{pmatrix}^k \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

for any integer \( k > 0 \).
5. Show that the following function

\[ \mathbf{x} \cdot \mathbf{y} = 3x_1 y_1 + 2x_2 y_2 \]

is an inner product on \( \mathbb{R}^2 \).
6. Let

\[ V = \text{span} \left( \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right). \]

Find an orthonormal basis for the orthogonal complement of \( V \).