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Math54 Sample Midterm I, Fall 2007

This is a closed book, closed notes exam. You need to justify every one of your answers unless you are asked not to do so. Completely correct answers given without justification will receive little credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so. Hand in this exam before you leave.

Problem	Maximum Score	Your Score
1	5	
2	19	
3	19	
4	19	
5	19	
6	19	
Total	100	

1. (5 Points)

   Your Name: ____________________________

   Your GSI: ____________________________

   Your SID: ____________________________

2. (19 Points)

   (a) Solve linear systems of equations $A\,x = b$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 2 & 1 & 1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 6 \\ 7 \\ 4 \end{pmatrix}.$$

   (b) Consider linear systems of equations $A\,x = b$, where

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & 2 & k^2 \\ 2 & 1 & 1 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 6 \\ 3k \\ 4 \end{pmatrix}.$$

   For what values of k does the system have a unique solution? infinite number of solutions? no solution?

3. (19 Points) Let $\mathcal{P}$ be the set of all functions of the form $c_0 + c_1 \sin(x)\cos(x) + c_2 \cos^2(x) + c_3 \sin^2(x)$, where the c's are arbitrary real constants. It is known that $\mathcal{P}$ is a linear space under the usual function addition and scalar multiplication. Find the dimension and a basis for $\mathcal{P}$.

4. (19 Points) Let $u_1, \cdots, u_m$ be vectors in $\mathbf{span}\{v_1, \cdots, v_k\}$; and let $v_1, \cdots, v_k$ be vectors in $\mathbf{span}\{w_1, \cdots, w_n\}$. Show that $u_1, \cdots, u_m$ are vectors in $\mathbf{span}\{w_1, \cdots, w_n\}$.

5. (19 Points) If the image of an $n \times n$ matrix A is $\mathbf{R}^n$, show that A must be invertible.

6. (19 Points) Find examples of $n \times n$ matrices A and B such that A, B are not invertible but $A + B$ is.