Math221: Matrix Computations

Homework #12, Do not hand in

- Let $A \in \mathbb{R}^{n \times n}$ be a square matrix. Let $H \in \mathbb{R}^{n \times n}$ be the upper Hessenberg matrix obtained from Hessenberg reduction on $A$. It is known that there exists an orthogonal matrix $Q$ such that $Q^T AQ = H$. What is the first column of $Q$?

- Let $A \in \mathbb{R}^{n \times n}$ be a square matrix. Let $Q$ be orthogonal and $H$ upper Hessenberg such that $Q^T AQ = H$. Let $P$ be the Householder matrix, the first column of which is the same as that of $Q$. Define $\hat{A} = P^T AP$, and let $\hat{H}$ be the upper Hessenberg matrix obtained from Hessenberg reduction on $\hat{A}$. What is the relationship between $\hat{H}$ and $H$?