Math221: Matrix Computations

Homework #10, Due Nov. 15, 2007

• Generate symmetric tridiagonal matrices of various dimensions, and run matlab codes `SQR.m` and `rayleigh.m` (available on class website). Demonstrate local cubic convergence on your matrices. Do Problem 5.13.

• Let $A = QΛQ^*$ be the eigendecomposition of $A$, with $Q = [q_1, \ldots, q_n]$, and let the initial vector $x_0 = q_1 + q_2$. Show that RQI fails to converge in exact arithmetic. Run `rayleigh.m` with this initial vector to see what it does in finite precision.

• Let $B \in \mathbb{R}^{n \times n}$ be an upper bidiagonal matrix. Find explicit formulas for its inverse.

• Generate upper bidiagonal matrices of various dimensions, and run matlab code `BiSVD.m` (available on class website) to compute their smallest singular values. You should try different scalings on the diagonal entries so the smallest singular values can be really tiny ($10^{-100} - 10^{-50}$, for example).

To check that these are indeed very accurate singular values, we use the formula

$$1/σ_{\text{max}} \left( B^{-1} \right) = σ_{\text{min}} \left( B \right). \quad (1)$$

The matlab `svd` function is backward stable. We generate $B^{-1}$ explicitly using the explicit formulas. This way the largest singular value of $B^{-1}$ is computed to full machine precision. Compare $1/σ_{\text{max}} \left( B^{-1} \right)$ with the singular values computed using `BiSVD.m` to show that `BiSVD.m` is highly accurate even for tiny singular values.

• Problems 5.27, 5.28.