Homework 1 Math 170

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Problem set: *Franklin* Section 1.2, Problems 1,2,3,5,6,9,10,14,15,16

Problems checked: *Franklin* Section 1.2, Problems 1,2,3,5,6,14,15

Grading scheme:

X	for "complete": significant effort demonstrated
O	for "fail": lack of demonstration of significant effort

Problems graded: *Franklin* Section 1.2, Problems 9,10,16

Grading scheme:

3	for "excellent":	Necessary steps are all shown and well explained. Solution is correct.
2	for "fair":	Necessary steps are all shown. There are minor gaps in explanantion and/or minor errors in solution.
1	for "poor":	Necessary steps are lacking. There are major gaps in explanantion and/or major errors in solution.
0	for "fail":	Significant effort is not demonstrated.

Sample solutions:

***Franklin* Section 1.2, Problem 9** Introducing slack variables $y_1, y_2, y_3, z_1, z_2, z_3$, all greater than or equal to zero, and writing $x_1 = u_1 - v_1$, $x_2 = u_2 - v_2$ for u_1, u_2, v_1, v_2 all greater than or equal to zero, the resulting linear program in canonical form is

$$\begin{bmatrix} 3 & 4 & -3 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 3 & 4 & -3 & -4 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 2 & 3 & -2 & -3 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 2 & 3 & -2 & -3 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 4 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 4 & 1 & -4 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ y_1 \\ y_2 \\ y_3 \\ z_1 \\ z_2 \\ z_3 \\ \varepsilon \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 5 \\ 5 \\ 9 \\ 9 \end{bmatrix}$$

$$\mathbf{u} \geq 0, \mathbf{v} \geq 0, \mathbf{y} \geq 0, \mathbf{z} \geq 0$$

$$\text{minimize} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \\ y_1 \\ y_2 \\ y_3 \\ z_1 \\ z_2 \\ z_3 \\ \varepsilon \end{bmatrix}$$

Since canonical form is not specified, it is also acceptable to give the solution in standard minimum form (see solution to **16** below).

***Franklin* Section 1.2, Problem 10** Introducing the new unknown $\mathbf{w} = (w_1, w_2, \ldots, w_6) \in \mathbb{R}^6$, the dual to the preceding is

$$\mathbf{w}^{\mathrm{T}} \begin{bmatrix} 3 & 4 & -3 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 3 & 4 & -3 & -4 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 2 & 3 & -2 & -3 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 2 & 3 & -2 & -3 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 4 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 4 & 1 & -4 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \leq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{maximize } \mathbf{w}^{\mathrm{T}} \begin{bmatrix} 7 \\ 7 \\ 5 \\ 5 \\ 9 \\ 9 \end{bmatrix}$$

Since canonical form for the preceding is not specified, it is also acceptable to give the solution as the dual to the standard minimum form (see solution to **16** below).

***Franklin* Section 1.2, Problem 16** Writing the problem in standard minimum form using block matrices,

$$\begin{bmatrix} -A \\ A \end{bmatrix} \mathbf{x} \geq \begin{bmatrix} -\mathbf{b} - \mathbf{d} \\ \mathbf{b} - \mathbf{d} \end{bmatrix}, \mathbf{x} \geq 0, \text{minimize } \mathbf{c}^{\mathrm{T}}\mathbf{x}$$

then, introducing the new unknown $\mathbf{y}$, the dual linear program is

$$\mathbf{y}^{\mathrm{T}} \begin{bmatrix} -A \\ A \end{bmatrix} \leq \mathbf{c}^{\mathrm{T}}, \mathbf{y} \geq 0, \text{maximize } \mathbf{y}^{\mathrm{T}} \begin{bmatrix} -\mathbf{b} - \mathbf{d} \\ \mathbf{b} - \mathbf{d} \end{bmatrix}$$

It is also possible to introduce slack variables to first write the expression in canonical form, then to give the dual to that (see solutions to **9**,**10** above).