Problem set: Franklin Section 1.17, Problems 1,2,3,4,5,6,9,11

Problems checked: Franklin Section 1.17, Problems 1,2,3,5,6

Grading scheme:

\[ X \] for “complete”: significant effort demonstrated
\[ O \] for “fail”: lack of demonstration of significant effort

Problems graded: Franklin Section 1.17, Problems 4,9,11

Grading scheme:

3 for “excellent": Necessary steps are all shown and well explained.
Solution is correct.

2 for “fair": Necessary steps are all shown.
There are minor gaps in explanation and/or minor errors in solution.

1 for “poor": Necessary steps are lacking.
There are major gaps in explanation and/or major errors in solution.

0 for “fail": Significant effort is not demonstrated.
Sample solutions:

**Franklin Section 1.17, Problem 4** Following example 2 in the text, start with

\[
\begin{bmatrix}
0 & 0 & 0 \\
3 & 3 & 3 & 2 \\
5 & 1 & 3 & 5 \\
3 & 2 & 1 & 3
\end{bmatrix}
\]

qualification matrix

\[
\begin{bmatrix}
1 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 1
\end{bmatrix}
\]

Complete assignment is impossible, so we need to continue. It is possible to increase \( v_3 \) and decrease \( u_2 \) and \( u_3 \) by one without violating the constraint, giving

\[
\begin{bmatrix}
0 & 0 & 1 \\
3 & 3 & 3 & 2 \\
4 & 1 & 3 & 5 \\
2 & 2 & 1 & 3
\end{bmatrix}
\]

qualification matrix

\[
\begin{bmatrix}
1 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 1
\end{bmatrix}
\]

Complete assignment is now possible, with the 2 in the first column, the 3 in the top spot of the second column and the 5 in the third column, giving a maximum sum of 10 for the OK subsets. This solution is readily apparent by inspection.

**Franklin Section 1.17, Problem 9** Clearly, for any \( c_1, c_2 \) in the network, the shortest path from \( a \) to \( c_2 \) is less than or equal to the shortest path from \( a \) to \( c_1 \) plus \( \tau(c_1, c_2) \), so \( \phi \) is an admissible potential. For any \( \lambda \neq \omega \), \( \phi(\omega) \leq \phi(\lambda) + \tau(\lambda, \omega) \), so this also holds for the minimizing \( \lambda \). On the other hand, for \( \lambda \) the preceding node to \( \omega \) on the minimizing path from \( a \) to \( \omega \), equality holds, so \( \phi(\omega) \) is greater than or equal to the minimum over all \( \lambda \neq \omega \). Hence, the functional equation holds.

**Franklin Section 1.17, Problem 11** Letting \( \otimes \) represent the Kronecker product of matrices (look this up if you are not familiar with it; it is a very handy thing to know), \( \text{Vec} \) be the operation that stacks the columns of a matrix to form a column vector, \( I_n \) be the \( n \times n \) identity matrix, and \( e_n \) be the row matrix of \( n \) ones, then the transportation problem can be written in block form as the canonical optimization problem

\[
\left[ e_m \otimes I_n \right] \text{Vec}X = \begin{bmatrix} s \\ d \end{bmatrix}, \text{Vec}X \geq 0, \text{minimize}(\text{Vec}C)^T \text{Vec}X
\]

with dual problem

\[
\left[ u^T \ v^T \right] \left[ e_m \otimes I_n \right] \leq (\text{Vec}C)^T, \text{maximize}\left[ u^T \ v^T \right] \begin{bmatrix} s \\ d \end{bmatrix}
\]

which written in indicial notation is what is asked to demonstrate.