1. **Problem 3:** Let

\[ A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = (a_1 \ a_2 \ a_3). \]

Then \( a_1, a_2, a_3, a_1, a_2, a_3, a_3, a_1 \) are the subsets that generate basic cones. The other set \( a_1, a_2, a_3 \) is linearly dependent.

2. **Problem 4:** After a change in any entry of \( A \), its columns become linearly independent. So all subsets, including \( a_1, a_2, a_3 \), generate basic cones.

3. **Problem 6:** Let \( C_1, \cdots, C_N \) be closed sets, and let \( C \) be their union. We want to show that \( C \) is closed. To this end, let \( \{x^n\}_{n=1}^{\infty} \) be a sequence in \( C \) with a limit,

\[ \lim_{n \to \infty} x^n = x^*. \]

We have to show that \( x^* \in C \).

Since we have a finite number of the closed sets \( C_i \)'s but infinite number of points in the sequence \( \{x^n\}_{n=1}^{\infty} \), it must be that there must be a specific closed set, say, \( C_t \), in which an infinite subset of \( x^n_{n=1}^{\infty} \) lives. Call this subset \( \{x^{n_k}\}_{k=1}^{\infty} \). This subset then is also convergent and converges to the same limit. Hence

\[ \lim_{k \to \infty} x^{n_k} = x^*. \]

Since \( C_t \) is a closed set, and the subsequence lives in it, it follows that \( x^* \in C_t \), hence \( x^* \in C \).

The word finite is necessary. Consider an infinite sequence of sets

\[ C_N = [1/N, 1], \]

for \( N = 1, 2, \cdots, \) .... These sets are all closed, but their union is \( (0, 1] \), which is not closed.

4. **Problem 7:** Since rank of \( A \) is \( r \), any subset of at least \( r + 1 \) columns is linearly dependent. But a subset of at most \( r \) columns could be linearly independent. Hence an upper bound on the total number of basic cones is

\[ \sum_{j=1}^{r} C_n^j. \]