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Math128B: Numerical Analysis Midterm

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	16	
2	18	
3	16	
4	16	
5	18	
6	16	
Total	100	

Your Name: ______________________________

Your SID: ______________________________

Your GSI: ______________________________

1. Compute the ∞-norm and 2-norm of the vector $\mathbf{v} = (1, 1, 1, 2, 3)^T$.

2. Verify that the function $\| \cdot \|_1$, defined on $\mathcal{R}^n$ by

$$\|\mathbf{x}\|_1 = \sum_{j=1}^{n} |x_j|$$

is a norm on $\mathcal{R}^n$.

3. A pair of vectors $\mathbf{u}$ and $\mathbf{v}$ are A-orthogonal if $\mathbf{u}^T A \mathbf{v} = 0$. Show that an A-orthogonal set of nonzero vectors for a symmetric positive definite matrix A is linearly independent.

4. Show that the functions $\phi_j(x) = \mathbf{sin}(j\,x), j = 1, \cdots, n$, are orthogonal on interval $[-\pi, \pi]$.

5. In the SVD of the matrix A:

$$A = U\,S\,V^T = (\mathbf{u}_1, \cdots, \mathbf{u}_m) \begin{pmatrix} s_1 & & \\ & \ddots & \\ & & s_n \\ 0 & \cdots & 0 \end{pmatrix} (\mathbf{v}_1, \cdots, \mathbf{v}_n)^T,$$

with $s_1 \geq s_2 \cdots \geq s_n \geq 0$ being the singular values of A. Show that the rank of A is the number of positive singular values.

6. Let $\mathbf{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Compute $\|\mathbf{u}\|_2$ and find the Givens rotation G so that

$$G\,\mathbf{u} = \begin{pmatrix} \|\mathbf{u}\|_2 \\ 0 \end{pmatrix}.$$