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Math128B: Numerical Analysis Practice Final

This is a closed book exam, but you are allowed a one-page cheat sheet. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly. You need not simplify your answers unless you are specifically asked to do so.

Problem	Maximum Score	Your Score
1	12.5	
2	12.5	
3	12.5	
4	12.5	
5	12.5	
6	12.5	
7	12.5	
8	12.5	
Total	100	

Your Name: ______________________________

Your SID: ______________________________

Your GSI: ______________________________

1. Show that the function

$$\|A\|_{\mathbf{max}} = \mathbf{max}_{i,j=1}^{n} |A_{i,j}|$$

is not a matrix norm.

2. Let

$$A = \begin{pmatrix} 1 & 0 & -1 \\ -\frac{1}{2} & 1 & -\frac{1}{4} \\ 1 & -\frac{1}{2} & 1 \end{pmatrix}.$$

Is A strictly diagonally dominant? Compute the spectral radius of the Gauss-Seidel matrix.

3. Show that if $\{\phi_0(x), \phi_1(x), \cdots, \phi_n(x)\}$ is an orthogonal set of functions on $[a, b]$ with respect to the weight function w, then $\{\phi_0(x), \phi_1(x), \cdots, \phi_n(x)\}$ is a linearly independent set.

4. Determine the trigonometric interpolating polynomial $S_2(x)$ of degree 2 on $[-\pi, \pi]$ for the function $f(x) = x\,(\pi - x)$.

5. Show that the following pair of matrices are not similar.

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}.$$

6. Let G be a Givens rotation $G = \begin{pmatrix} c & s \\ -s & c \end{pmatrix}$ with $c^2 + s^2 = 1$. Show that G can not be a Householder matrix. In other words, there does not exist a unit vector $\mathbf{v}$ such that $G = I - 2\mathbf{v}\,\mathbf{v}^T$.

7. Use Newton's method with $\mathbf{x}^{(0)} = \mathbf{0}$ to compute $\mathbf{x}^{(1)}$ for the following nonlinear system

$$\begin{aligned} x_1(1 - x_1) + 4x_2 &= 12, \\ (x_1 - 2)^2 + (2x_2 - 3)^2 &= 25. \end{aligned}$$

8. Show that if $\mathbf{u}, \mathbf{v} \in \mathcal{R}^n$ then $\mathbf{det}\left(I + \mathbf{u}\,\mathbf{v}^T\right) = 1 + \mathbf{v}^T\,\mathbf{u}$.