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Math128B: Numerical Analysis Sample Midterm

This is a closed book, closed notes exam. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly.

Problem	Maximum Score	Your Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Your Name: _____

Your SID: _____

1. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$.

    (a) Compute the spectral radius of A.

    (b) For the linear system of equations $Ax = b$ where $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, define the Jacobi method.

    (c) Find out whether the Jacobi method converges for any initial guess x_0.

2. Let $f(x)$ be a continuous function on $[-1, 1]$. Define set

$$\mathcal{S} = \{P(x), \text{where } P(x) = \alpha_0 + \alpha_1 x + \cdots \alpha_n x^n, \text{ and } \alpha_1, \cdots, \alpha_n \text{ are real constants.}\}$$

Consider the least squares problem

$$\min_{P(x) \in \mathcal{S}} \int_0^1 \frac{1}{\sqrt{x}} \left(f(x) - P(x) \right)^2 dx.$$

Find the normal equation that defines the optimal $P(x)$.

3. Consider the set of equations

$$x_1(1 - x_1) + 4x_2 = 12,$$
$$(x_1 - 2)^2 + (2x_2 - 3)^2 = 25.$$

Find the Jacobian matrix and define the Newton iteration for these equations.

4. For a given $m \times n$ matrix $A = (a_{ij})$, define a matrix norm $\|A\|_{2,1}$ as

$$\|A\|_{2,1} = \max_{x \neq 0} \frac{\|Ax\|_2}{\|x\|_1}.$$

Partition $A = (a_1, a_2, \cdots, a_n)$, where a_1, a_2, $\cdots$, a_n are the columns of A. Show that

$$\|A\|_{2,1} = \max_{1 \leq j \leq n} \|a_j\|_2.$$