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Math128A: Sample Numerical Analysis Midterm

This is a closed book, closed notes exam, except a one-page/one-side cheatsheet. You need to justify every one of your answers. Completely correct answers given without justification will receive little credit. Do as much as you can. Partial solutions will get partial credit. Look over the whole exam to find problems that you can do quickly.

Problem	Maximum Score	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Your Name: _____

Your SID: _____

Your GSI: _____

1. Let

$$f(x) = \frac{e^x - 1}{x}.$$

- (a) Find $\lim_{x \rightarrow 0} f(x)$.
- (b) Approximate $f(x)$ by replacing the exponential function with its Maclaurin polynomial of degree 3 in $f(x)$.
- (c) Provide an upper bound on the approximation error for $x \in [-1, 1]$. You can leave your upper bound as a computable expression.

2. A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \leq x \leq 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \leq x \leq 2. \end{cases}$$

Find b, c and d .

3. The quadrature formula

$$\int_{-1}^1 f(x) dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$$

is exact for all polynomials of degree less than or equal to 2.

- (a) Determine c_0 , c_1 and c_2 .
- (b) What is the degree of precision of this quadrature?

4. Let $x_0 < x_1 < x_2$. Show that there is a unique polynomial $P(x)$ of degree at most 3 such that

$$P(x_j) = f(x_j), \quad j = 0, 1, 2, \quad \text{and} \quad P'(x_1) = f'(x_1).$$

Give an explicit formula for $P(x)$.

5. Let $f(x) = x^2 - 2$.

- (a) Given $p_0 = 0$, use Newton's method to find p_1 .
- (b) Show that $f(x)$ has a root in $[0, 2]$.
- (c) Given the initial interval $[0, 2]$, perform one step of bisection.