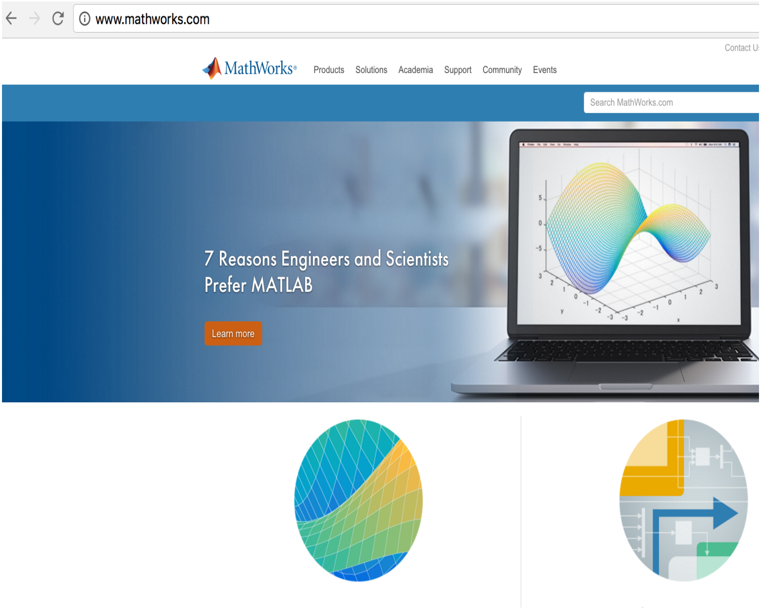
Self Introduction

- **Name:** Ming Gu
- **Office:** 861 Evans
- **Email:** mgu@berkeley.edu
- **Office Hours:** TuWTh 1:00-2:30PM
- **Class Website:**
  math.berkeley.edu/∼mgu/MA128AFall2017

Text Book

- Burden and Faires, **Numerical Analysis**.
  Required. 9^{th} edition recommended.

Matlab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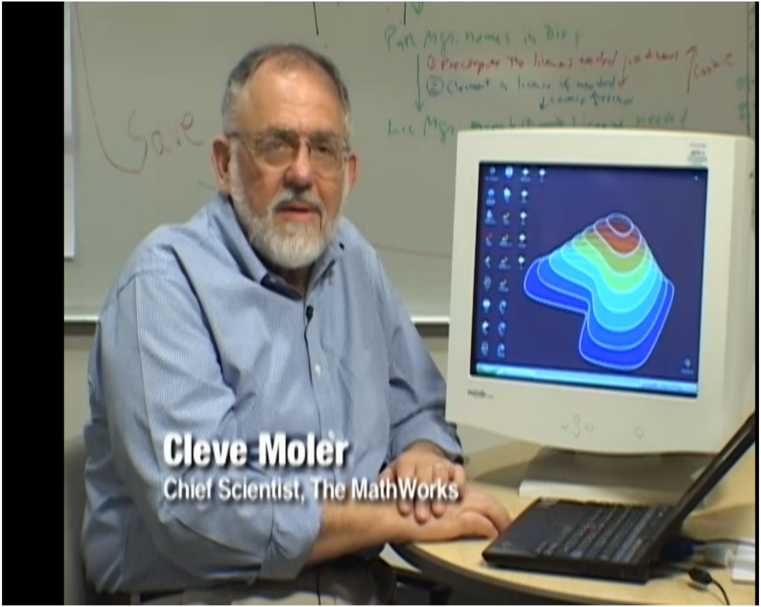
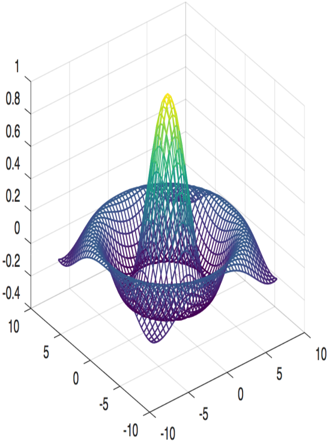


and maybe Octave

s://www.gnu.org/software/octave/

GNU Octave   About   Donate   Download   Get Involved   News   Support/Help   Docs



GNU Octave

Scientific Programming Language

- Powerful mathematics-oriented syntax with built-in plotting and visualization tools
- Free software, runs on GNU/Linux, macOS, BSD, and Windows
- Drop-in compatible with many Matlab scripts

Download | Docs

Math 98: Introduction to Matlab

- runs 6 weeks, starting next week;
- two sections, 30 students each;
- some slots are still available;
- **Math 98 Website:**
  math.berkeley.edu/∼myzhang/teaching.html#

Class Work

- Up to 13 weekly home work sets;
  Count best 10, total 10 points.
- 5 Quizzes;
  Count best 4, total 10 points.
- 2 Programming Assignments, total 20 points;
- 1 Midterm exam, 25 points;
- 1 Final exam, 35 points.
- Final worth 60 points if Midterm missing.

Quiz and Exam Schedule

- **Quiz**: Sept. 6 in discussion
- **Quiz**: Sept. 20 in discussion
- **Programming Assignment 1**: 11:59PM, Sept. 27
- **Quiz**: Oct. 4 in class
- **Midterm**: Oct. 18 in class
- **Quiz**: Nov. 1 in discussion
- **Quiz**: Nov. 15 in discussion
- **Programming Assignment 2**: 11:59PM, Nov. 29
- **Final Exam**: December 14, 8:00-11:00AM (Exam Group 13)

Grade Scale

- **A**- to **A+**: at least 85 points;
- **B**- to **B+**: between 70 and 85 points;
- **C**- to **C+**: between 60 and 70 points;
- **D**: between 55 and 60 points;
- **F**: less than 55 points.

No grade curve; most people get A level or B level grades.

Numerical Analysis = Calculus on a computer

- First 6 Chapters of Text Book.
- **Chapter 1**: Calculus Review, Computer Math.
- **Chapter 2**: Solve $f(x) = 0$.
- **Chapter 3**: Approximate given functions.
- **Chapter 4**: Numerical derivatives, integrals.
- **Chapter 5**: Numerical Initial value ODEs.
- **Chapter 6**: Solve linear equations $Ax = b$.

Fibonacci's Problem in 1224, with Emperor Frederick II

Solve

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0.$$

Fibonacci's Solution

$$x = 1+22\left(\frac{1}{60}\right)+7\left(\frac{1}{60}\right)^2+42\left(\frac{1}{60}\right)^3+33\left(\frac{1}{60}\right)^4+4\left(\frac{1}{60}\right)^5+40\left(\frac{1}{60}\right)^6.$$

There can only be ONE real solution since

$$f'(x) = 3x^2 + 4x + 10 = 3(x + 2/3)^2 + \frac{26}{3} > 0,$$

$f(x)$ is a monotonically increasing function.

Fibonacci's Problem in 1224, with Emperor Frederick II

Solve

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0.$$

Fibonacci's Solution

$$x_{\textbf{Fibon}} = 1+22\left(\frac{1}{60}\right)+7\left(\frac{1}{60}\right)^2+42\left(\frac{1}{60}\right)^3+33\left(\frac{1}{60}\right)^4+4\left(\frac{1}{60}\right)^5+\mathbf{40}\left(\frac{1}{60}\right)^6$$

The computer has a better solution

$$x_{\textbf{Comp}} = 1+22\left(\frac{1}{60}\right)+7\left(\frac{1}{60}\right)^2+42\left(\frac{1}{60}\right)^3+33\left(\frac{1}{60}\right)^4+4\left(\frac{1}{60}\right)^5+\mathbf{39}\left(\frac{1}{60}\right)^6$$


Mongol Empire
1190-1400
Fibonacci
1224
RUSSIAN PRINCIPALITIES
Kiev
1240
Volga
Danube
MEDITERRANEAN SEA
BLACK SEA
Constantinople
SELJUKS OF RUM
Konya
Lesser Armenia
Caucasus
Tiflis
Old Sarai
CASPIAN SEA
Urgench
ARAL SEA
Aleppo
Tabriz
Maragha
Sultaniyya
Alamut
Damascus
Acre
Jerusalem
1260
1258
MAMLUK EMPIRE
Baghdad
CALIPH'S DOMINIONS
Hamadan
Ravy
KHWARAZM
Nishapur
Marv
Balkh
Harat
Bukhara
Samarqand
Utrar
SHAH'S EMPIRE
Isfahan
Shiraz
ILKHANATE
Kirman
Nile
Tigris
Euphrates
Oxus
Hindu Kush
Indus
1219
1220
1221
1224
1236
1218
CHAGHATAI KHANATE
Lake Balkhash
Balasaghun
QARA-KHITAI
Kashgar
UIGHURS
MONGOL TRIBES
(unified 1206)
Karakorum
Lake Baikal
Gobi
1209
1226-7
1211
1215
Zhongdu (Beijing)
XIXIA
Shazhou
Ningxia
JIN EMPIRE
KHANS
Yellow R.
Kaifeng
Hangzhou
SONG EMPIRE
Quangzhao
Guangzhao
GREAT
Conquest of Sung
Text
Text
Text
TIBET
Yangtze
Brahmaputra
Delhi
DELHI SULTANATE
Ganges
Pagan
BURMA
ANNAM
KHMER EMPIRE
CHAMPA
Mekong
Approximate boundary of Empire at death of Genghis Khan 1227
Approximate boundary of Empire c. 1280
Boundaries of Mongol Khanates
Campaigns under Genghis Khan 1206–27
Later major Mongol campaigns
KHAN Mongol Khanates within the Empire
TIBET Pre-Mongol states

Fibonacci's Cubic Root is correct to 11 digits

```
>> format long e;
>> h = [1 2 10 -20];
>>  r = roots(h)

r =

    -1.684404053910685e+00 + 3.431331350197691e+00i
    -1.684404053910685e+00 - 3.431331350197691e+00i
     1.368808107821373e+00

>> Fibonacci = (((((40/60+4)/60+33)/60+42)/60+7)/60+22)/60+1

Fibonacci =

    1.368808107853224e+00

>>  r(3)-Fibonacci

ans =

   -3.185118835347112e-11

>> Better = ((((((31/60+38)/60+4)/60+33)/60+42)/60+7)/60+22)/60+1

Better =

    1.368808107821430e+00

>> r(3)-Better

ans =

   -5.795364188543317e-14
```

Roots of a Random Quintic Polynomial: No closed form formula

```
>>  format short g
>>  hrand = randn(1,6)

hrand =

     -1.3499       3.0349       0.7254    -0.063055      0.71474     -0.20497

>> rrand = roots(hrand)

rrand =

      2.4872
    -0.70735
       0.105 +    0.56831i
       0.105 -    0.56831i
      0.2584
```

Numerical Methods can find roots for any polynomial

Simple Numerical Integration (I)

- Some integrals are easy to integrate by hand

$$I_1 = \int_{-1}^{1} \sqrt{1+x}\, dx = 4\sqrt{2}/3.$$

- But others are harder

$$I_2 = \int_{-1}^{1} \sqrt{1+x^2}\, dx = ?.$$

Numerical Methods attempt to numerically evaluate any integral

Simple Numerical Integration (I): 6 digits correct

```
>> format long g
>> I2 = quad(@(x) sqrt(1+x.^2), -1 ,1 )

I2 =

          2.29558701441275

>> I1 = quad(@(x) sqrt(1+x), -1 ,1 )

I1 =

          1.88561089016424

>> I1 - (4/3)*sqrt(2)

ans =

     -7.19299988971578e-06
```

Simple Numerical Integration (II): 12 digits correct

```
>> I1tol = quad(@(x) sqrt(1+x), -1 ,1,1e-12 )

I1tol =

          1.88561808316058

>> I1tol - (4/3)*sqrt(2)

ans =

      -3.5500491435414e-12
```

Simple Numerical Integration: can't go overboard in accuracy

```
>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-4);disp([I1-(4/3)*sqrt(2),fcnt1])
  -4.5143e-04   2.1000e+01

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-8);disp([I1-(4/3)*sqrt(2),fcnt1])
  -4.0728e-08   1.1300e+02

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-12);disp([I1-(4/3)*sqrt(2),fcnt1])
  -3.5500e-12   6.9700e+02

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-16);disp([I1-(4/3)*sqrt(2),fcnt1])
  -4.4409e-16   4.3930e+03

>> [I1,fcnt1] = quad(@(x) sqrt(1+x), -1 ,1, 1e-20);disp([I1-(4/3)*sqrt(2),fcnt1])
Warning: Maximum function count exceeded; singularity likely.
> In quad at 106
  -5.6018e-06   1.0017e+04
```

Trajectory with ODE

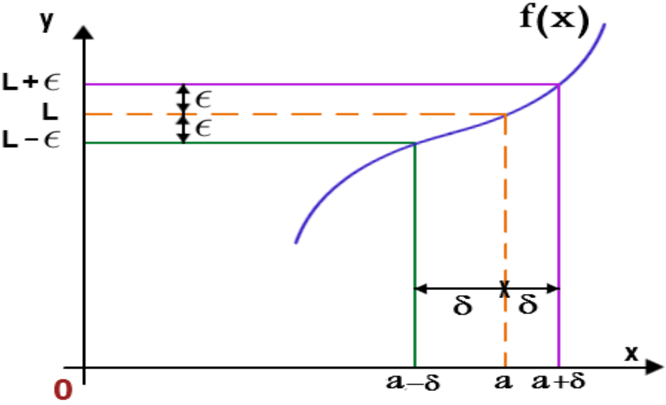
Problem to solve: find function $y(t)$ so that

$$y'(t) = f(y(t), t), \quad y(t_0) = y_0$$

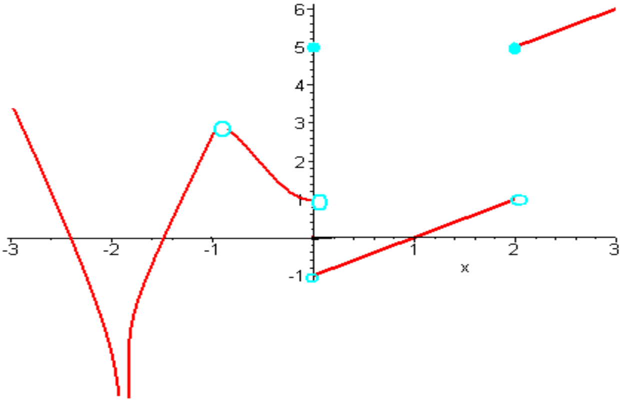
$y(t)$ could be trajectory of a flying bullet, y_0 is initial position.

trajectory follows ODE, aiming is setting initial condition

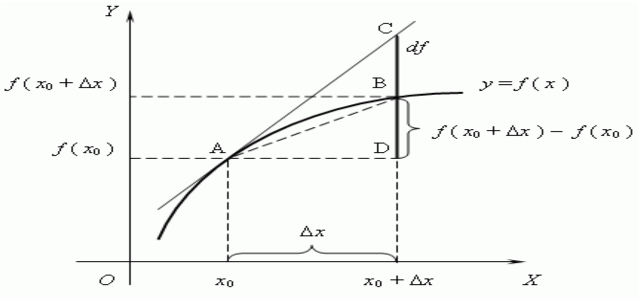
Calculs Review: Limit



Continuity vs. Discontinuity

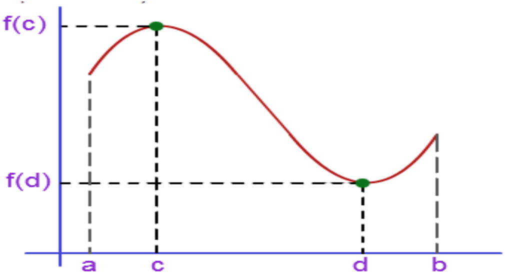


Def: Differentiability



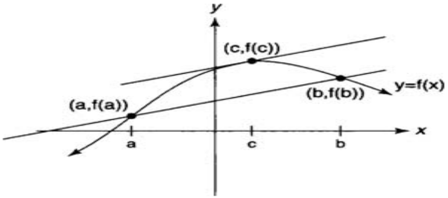
$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

Extreme Value Theorem



- Maximum $f(c)$ and minimum $f(d)$ attainable in $[a, b]$ if $f(x)$ continuous.
- Basis of much of data analysis, artificial intelligence.

Mean Value Theorem

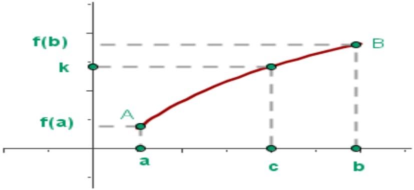


- If $f(x)$ continuous, then c exists in $[a, b]$ so

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

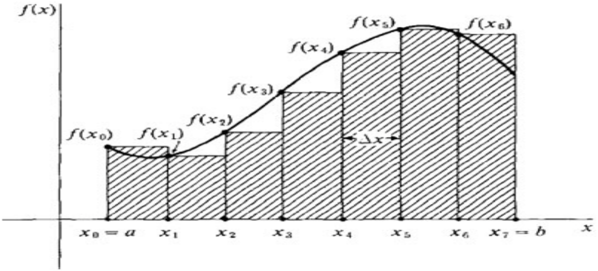
- Basis of much of theoretical analysis.

Intermediate Value Theorem



- If $f(x)$ continuous, then c exists in $[a, b]$ so $f(c) = k$ for any k between $f(a)$ and $f(b)$.
- Basis of methods for solving $fx) = 0$.

Riemann Sum



- $$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f(x_k).$$