Math128A: Numerical Analysis Sample Midterm I

1. (40 Points) Consider the iteration

\[ x_{n+1} = \frac{x_n^3 + 3ax_n}{3x_n^2 + a}. \]

(a) What is it intended to compute?
(b) Given \( a = 2 \) and \( x_0 = 1 \), compute \( x_1 \) and \( x_2 \).
(c) Define and determine the order of convergence of this iteration.

2. (30 Points)

(a) For a function \( f \) and distinct points \( a, b \) and \( c \), define what is meant by \( f[a,b,c] \).
(b) Find the Lagrange form of the polynomial \( p \) which interpolates \( f(x) = 4x/(x + 1) \) at \( 0, 1 \) and \( 3 \).
(c) Find \( f[0, 1, 3] \).

3. (30 Points) Find the maximum and minimum values of the function \( f(x) = \sin(x) - \cos(x) + 1 \) on the interval \([\pi, \pi]\).