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Math110 Sample Midterm I, Fall 2011

This is a closed book exam; but everyone is allowed a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers unless you are asked not to do so. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so. Hand in this exam before you leave.

Problem	Maximum Score	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Write your personal information below and on top of every page in the test.

Your Name: ______________________________

Your GSI: ______________________________

Your SID: ______________________________

1. Let $\mathcal{P}$ be the set of all polynomials with real coefficients.

   (a) Let $\mathcal{Q}_3$ be the set of all polynomials of degree exactly 3 with real coefficients. Is $\mathcal{Q}_3$ a subspace of $\mathcal{P}$?

   (b) Let $\mathcal{R}_3 = \{a(1+x^2) + b(x-3^2) \mid \quad a \text{ and } b \text{ are real numbers.}\}$. Is $\mathcal{R}_3$ a subspace of $\mathcal{P}$?

2. Let

$$f_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad f_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad f_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \quad f_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Show that f_1, f_2, f_3, f_4 are linearly independent.

3. Let

$$S = \{(1,2,3),(2,3,4),(3,4,5)\}.$$

Find the dimension and a basis for $\textbf{span}(S)$.

4. Define a function $T : \mathbf{R}^2 \to \mathbf{M}_{2\times 2}(\mathbf{R})$ as

$$T\,(\,a_1, a_2\,) = \begin{pmatrix} a_1 & a_2 \\ 2a_2 & a_1 \end{pmatrix}.$$

   (a) Show that T is a linear transformation.

   (b) Find bases for its range and null spaces.

5. Find linear transformations $\mathbf{U}, \mathbf{T} : \mathbf{R}^2 \to \mathbf{R}^2$ such that $\mathbf{UT} = \mathbf{T_0}$ (the zero transformation) but $\mathbf{TU} \neq \mathbf{T_0}$.