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Math110 Sample Final, Fall 2011

This is a closed book exam; but everyone is allowed a standard one-page cheat sheet (on one-side only). You need to justify every one of your answers unless you are asked not to do so. Completely correct answers given without justification will receive little credit. Problems are not necessarily ordered according to difficulties. You need not simplify your answers unless you are specifically asked to do so. Hand in this exam before you leave.

Problem	Maximum Score	Your Score
1	20	
2	20	
3	15	
4	15	
5	15	
6	15	
Total	100	

Write your personal information below and on top of every page in the test.

Your Name: ______________________

Your GSI: ______________________

Your SID: ______________________

1. Label each of the following statements as **TRUE** or **FALSE**. Along with your answer, provide a counterexample, an informal proof or an explanation.

   (a) Every normal operator is diagonalizable.

(b) Similar matrices always have the same eigenvectors.

(c) The pseudoinverse of any linear operator exists even if the operator is not invertible.

2. Label each of the following statements as **TRUE** or **FALSE**. Along with your answer, provide a counterexample, an informal proof or an explanation.

   (a) If $(A \mid b)$ is in reduced row echelon form, then the system $Ax = b$ is consistent.

(b) Every change of coordinate matrix is invertible.

(c) An elementary matrix is always square.

3. Let $\mathbf{P}_3(R)$ and $\mathbf{P}_4(R)$ be the sets of real polynomials of degrees at most 3 and 4, respectively. Define

$$\mathbf{T} : \mathbf{P}_3(R) \to \mathbf{P}_4(R) \quad \text{by} \quad \mathbf{T}(f(x)) = \int_0^x f(t)dt.$$

Prove that $\mathbf{T}$ is linear, and find a basis for the range space of $\mathbf{T}$.

4. Let A and B be $m \times n$ matrices. Show that

$$\mathbf{rank}(A+B) \leq \mathbf{rank}(A) + \mathbf{rank}(B).$$

5. (a) Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Find the SVD of A.

(b) Proe that if A is a symmetric positive definite matrix and $A = U\Sigma V^T$, then $U = V$.

6. Let V be an inner product space, and let $S = \{v_1, v_2, \cdots, v_n\}$ be an orthonormal subset of V. Prove that for any $x \in V$ we have

$$\|x\|^2 \geq \sum_{i=1}^{n} |< x, v_i >|^2.$$