1. Recall some properties of logarithms:

$$\log_a b^n = n \log_a b$$
$$\log_a(bc) = \log_a b + \log_a c$$
$$\log_a(b/c) = \log_a b - \log_a c$$
$$\frac{\log_a b}{\log_a c} = \log_c b$$

Here are some exercises to review these properties (and some of the properties of exponents). Compute the following (simplify as much as possible):

(a) $(2^{4x} \cdot 2^{-x})^{1/2}$
Solution: $2^{3x/2}$

(b) $8^{x/3} \cdot 16^{3x/4}$
Solution: $= 2^{3x/3} \cdot 2^{3 \cdot 4x/4} = 16^x$

(c) $\ln 4 + 2 \ln 3$
Solution: $= \ln 4 + \ln 9 = \ln 36$

(d) $\log_2(3) \cdot \log_3(4)$
Solution: $= \ln(2)/\ln(3) \cdot \ln(3)/\ln(4) = \ln(2)/\ln(4)$

(e) $6^a - 6^b$
Solution: $= 6^a - 6^b$

(f) $\log_3(100) - \log_3(10)$
Solution: $= \log_3(100/10) = \log_3(10)$

2. Differentiate the following with respect to x:

(a) $f(x) = x^2 e^x$
Solution: $f'(x) = e^x(2x + x^2)$

(b) $f(x) = e^{1/x}$
Solution $f'(x) = -\frac{1}{x^2} e^{1/x}$

(c) $f(x) = \sqrt{1 + xe^{-2x}}$
Solution $f'(x) = 1/2 \cdot (1 + xe^{-2x})^{-1/2} \cdot (1 - 2x)e^{-2x}$

(d) $f(x) = e^{e^{e^x}}$
Solution:

$$f'(x) = e^{e^{e^x}} \cdot e^{e^x} \cdot e^x = e^{x + e^x + e^{e^x}}$$

(e) $f(x) = \ln(x^2 + 2)$
Solution: $f'(x) = 2x/(x^2 + 2)$

(f) $f(x) = \ln(x + \sqrt{x^2 - 1})$
Solution: $1/(x + \sqrt{(}x^2 - 1)) \cdot (1 + 1/2 \cdot (x^2 - 1)^{-1/2} \cdot (2x))$

(g) $f(x) = \ln(\ln(\ln x))$
Solution:

$$f'(x) = 1/(\ln(\ln(x)) \cdot (1/\ln(x)) \cdot (1/x) = \frac{1}{x \ln(x) \ln(\ln(x))}$$

(h) $f(x) = \frac{x^2\sqrt{x^2+1}}{(3x+2)^5}$
Solution:

$$= \frac{(2x\sqrt{x^2+1} + x^3/\sqrt{x^2+1})(3x+2)^5 - (x^2\sqrt{x^2+1}) \cdot 15(3x+2)^4}{(3x+2)^{10}}$$

(i) Find the equation of the tangent line to the curve $y = \ln(x^3 - 7)$ at the point $(2, 0)$. **Solution:** Slope $y' = 3x^2/(x^3 - 7)$, evaluate at $x = 2$ to find $y'(2) = 12$ and thus equation of line is $y = 12 \cdot (x - 2) + 0$.

3. A certain population of bacteria grows at a rate proportional to its size. At 12:00 (noon) the population is 30000, while at 1pm the population is 40000. Calculate the size of the population at 3pm.

   Solution: We have $y(t) = Ce^k t$. Consider noon as being $t = 0$. Use $y(0) = 30000$ and $y(1) = 40000$, solve for C and k. The population at 3pm will be $y(3) = 30000(4/3)^3$.

4. If dairy cows eat hay containing too much iodine 131, their milk will be unfit to drink. If the hay contains 10 times the maximum allowable level of iodine 131, how many days should the hay be stored before it is fed do dairy cows? (Information: the half-live of iodine 131 is 8 days).

   Soultion: $t = 8\frac{\ln 10}{\ln 2}$.