Qualifying exam syllabus

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1 Major Topic: Numerical Methods for Differential Equations (Applied Mathematics)

1.1 Ordinary Differential Equations

1. Basic notions: convergence, consistency, stiff problem, A-, A(α)-, B-stability, region of absolute stability, local truncation error.

2. Runge-Kutta Methods: order conditions, stability, Butcher array, collocation and implicit method, Gaussian quadrature, error estimates and step size control, embedded Runge-Kutta.

3. Multistep methods: order conditions, zero-stability, Adams-Bashforth and Adams-Moulton, BDF.

1.2 Partial Differential Equations

1. Finite Differences: stability, consistency, convergence, local truncation error, CFL condition, von Neumann stability, basic schemes (upwinding, leapfrog, Lax-Wendroff, Lax-Friedrichs, Crank-Nicholson, ADI).

2. Hyperbolic Conservation Laws: Burgers' equation, conservation form, Glimm's and Godunov's methods.

3. Finite Elements: definition, weak formulation, discontinuous Galerkin method.

References :

- Randall J. LeVeque. Finite Difference Methods for Ordinary and Partial Differential Equations. SIAM, 2007.

- Per-Olof Persson. Lecture Notes from Math 228 A, Fall 2011.

- James A. Sethian. Lecture Notes from Math 228 B, Spring 2012.

2 Major Topic: Partial Differential Equations (Classical Analysis)

1. Laplace Equation: fundamental solution, mean value formulas, maximum principle, uniqueness, regularity, Harnack's inequality, Green's function, energy method.

2. Heat Equation: fundamental solution, mean-value formula, maximum principle, uniqueness, regularity, energy method.

3. Wave Equation: solution in 1-, 2-, and 3-D, method of spherical means, energy method, non-homogenous problem.

4. Characteristics: method of characteristics, local existence theorem.

5. Introduction to Hamilton Jacobi equations: Euler-Lagrange equation, Hamilton's ODE, Hopf-Lax formula, weak solutions.

6. Conservation Laws: shocks, entropy condition, Rankine-Hugoniot condition, Lax-Oleinik formula.

7. Sobolev Spaces: definition and properties, weak derivates, approximation by smooth functions, traces, Gagliardo-Nirenberg-Sobolev inequality, Poincare's inequality.

8. Second Order Elliptic Equations: definition and weak solutions, existence theorems, Lax-Milgram theorem, Fredholm alternative, maximum principles, eigenvalues and eigenfunctions.

Reference: Lawrence C. Evans. Partial Differential Equations. American Mathematical Society, 2010.

3 Minor Topic: Ordinary Differential Equations (Classical Analysis)

1. General theory: existence, uniqueness, extension of solutions

2. Linear systems: fundamental matrix, inhomogeneous problem, constant coefficients case, Floquet theory.

3. Boundary value problems: Green's functions, Sturm-Liouville theory, eigenvalue problems.

Reference: Earl A. Coddington and Norman Levinson. Theory of Ordinary Differential Equations, Krieger Pub Co.