Math 16A; Sample Second Midterm

Name: Solution

GSI:
Section number:
or time and room:

(do not write here)

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Please show all your work and exhibit your final answers clearly. You may use the backs of these pages for your extra work. You have 50 minutes.

Problem 1 (20 points)

A company makes a product. It costs 10 dollars to make one item. If it charges 30 dollars per item it will sell exactly 1,000 items; but if it charges 20 per item it will sell exactly 2,000 items. Assuming a linear demand function, how many items should the company make to maximize its profit.

Objective : Maximize Profit

Objective Equation:

Profit = Revenue − Cost

= price · quantity − Cost

Let p = price per item
x = number of items
C = total cost
P = total profit

$$P = xp - C$$

Constraints

$$C = 10x$$

if $p = 30$ then $x = 1000$
$p = 20$ then $x = 2000$

$$m = \frac{p_2 - p_1}{x_2 - x_1} = \frac{20-30}{2000-1000} = \frac{-10}{1000} = \frac{-1}{100}$$

$$p - 30 = \frac{-1}{100}(x - 1000) \Rightarrow p = \frac{-1}{100}x + 40$$

Objective Function

$$P(x) = x\left(-\frac{1}{100}x + 40\right) - 10x$$

$$= \frac{-1}{100}x^2 + 30x$$

Calculus

$$P'(x) = \frac{-1}{50}x + 30$$

$$0 = \frac{-1}{50}x + 30$$

$$\frac{1}{50}x = 30$$

$$x = \boxed{1500 \text{ items}}$$

Note $P''(x) = -\frac{1}{50} < 0$ everywhere so any local extrema is a max (because f is concave down)

Problem 2 (20 points)

$f(x)$ is a function whose derivative is $\sqrt{x^2+1}$. What is the derivative of $f(3x+2)$.

$$\frac{d}{dx}\left[f(3x+2)\right] = f'(3x+2)\cdot(3x+2)'$$
$$= \sqrt{(3x+2)^2+1}\cdot 3$$
$$= \boxed{3\sqrt{(3x+2)^2+1}}$$

Problem 3 (20 points)

y and z are functions of x with the property $z^3y^5 + z^5y^7 = 2$. Also, when $x=3$ we have $y=1$ and $z=1$. Also, when $x=3$, we have $\frac{dy}{dx}=5$. Find $\frac{dz}{dx}$ when $x=3$.

$$z^3y^5 + z^5y^7 = 2$$
$$\frac{d}{dx}\left[z^3y^5+z^5y^7\right] = \frac{d}{dx}[2]$$
$$\left(3z^2\frac{dz}{dx}y^5 + z^3\cdot 5y^4\frac{dy}{dx}\right) + \left(5z^4\frac{dz}{dx}\cdot y^7 + z^5\cdot 7y^6\frac{dy}{dx}\right) = 0$$
$$\frac{dz}{dx}\left[3z^2y^5 + 5z^4y^7\right] = -\frac{dy}{dx}\left[5z^3y^4 + 7z^5y^6\right]$$
$$\frac{dz}{dx} = -\frac{dy}{dx}\left[\frac{5z^3y^4 + 7z^5y^6}{3z^2y^5+5z^4y^7}\right]$$
$$= -\frac{dy}{dx}\left[\frac{5z + 7z^3y^2}{3y+5z^2y^3}\right]$$
$$\left.\frac{dz}{dx}\right|_{\substack{x=3\\y=1\\z=1}} = -5\left[\frac{5+7}{3+5}\right] = \boxed{\frac{-15}{2}}$$

Problem 4 (20 points)
Solve for x. (your answers can involve e and/or ln)

(a) (10 points) $e^{(3x+2)} = 5$

(b) (10 points) $\ln(3x^2) = 5$

(a) $e^{(3x+2)} = 5$

$\ln e^{3x+2} = \ln 5$

$3x+2 = \ln 5$

$3x = \ln 5 - 2$

$\boxed{x = \frac{\ln 5 - 2}{3}}$

(b) $\ln(3x^2) = 5$

$e^{\ln(3x^2)} = e^5$

$3x^2 = e^5$

$x^2 = \frac{e^5}{3}$

$\boxed{x = \pm\sqrt{e^5/3}}$

Problem 5 (20 points)
Find the derivative of $(x^2+1)^x$

$y = (x^2+1)^x$

$= \left(e^{\ln(x^2+1)}\right)^x$

$= e^{x\ln(x^2+1)}$

$y' = e^{x\ln(x^2+1)} \cdot [x\ln(x^2+1)]'$

$= (x^2+1)^x \left[\ln(x^2+1) + x \cdot \frac{1}{x^2+1} \cdot 2x\right]$

$= \boxed{(x^2+1)^x \left[\ln(x^2+1) + \frac{2x^2}{x^2+1}\right]}$