**Math 16A; Sample First Midterm**

(do not write here)

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Name:

GSI:
Section number:
or time and room:

Please show all your work and exhibit your final answers clearly. You may use the backs of these pages for your extra work. You have 50 minutes. CALCULATORS ARE NOT ALLOWED DURING THE EXAM

*Problem 1* (20 points)

(a) (10 points) Find the slope of the line through the points $(3, 2)$ and $(6, 1)$.

(b) (10 points) $f(x)$ is a function where $f(5) = 1$ and $f'(5) = -8$. Find the equation of the tangent line to $y = f(x)$ at $x = 5$.

(a)

$$\frac{2-1}{3-6} = \frac{-1}{3}$$

(b) $y = 1 + (-8)(x - 5)$

*Problem 2* (20 points)

(a) (10 points) What is the limit definition of $f'(5)$ where $f(x) = \frac{1}{x^2}$

$$limit_{h \to 0} \frac{(1/(5+h)^2) - (1/5^2)}{h}$$

(b) (10 points) Find a function $f(x)$ and a number $a$ so that the following is the limit definition of $f'(a)$

$$limit_{h \to 0} \frac{\sqrt{((3+h)^2 + 7} - 4}{h}$$

$$a = 3; f(x) = \sqrt{x^2 + 7}$$

*Problem 3* (20 points)

Let $f(x) = (1+5x)^3 + \sqrt{2+7x} + (1/(8+3x))$

(a) (10 points) Find the derivative of $f(x)$

(b) (10 points) find the second derivative of $f(x)$.

$f(x) = (1+5x)^3 + (2+7x)^{(1/2)} + (8+3x)^{(-1)}$

$f'(x) = 15(1+5x)^2 + (7/2)(2+7x)^{(-1/2)} + (-3)(8+3x)^{(-2)}$

$f''(x) = 150((1+5x) + (-49/4)(2+7x)^{(-3/2)} + (18)(8+3x)^{(-3)}$

*Problem 4* (20 points)

Find $f'(x)$ where $f(x) = \sqrt{5+(1+3x^2)^3}$

$f(x) = (5+(1+3x^2)^3)^{(1/2)}$

$f''(x) = (1/2)(5+(1+3x^2)^3)^{(-1/2)})(3(1+3x^2)^2)(6x)$

*Problem 5* (20 points)

Sketch the graph of $y = 5x^6 - 6x^5$.

(Be sure to **clearly** indicate where your graph is increasing, decreasing, concave-up, and concave-down).

$y = 5x^6 - 6x^5$
$y' = 30x^5 - 30x^4$
$y'' = 30(5x^4 - 4x^3)$

$y' = 30x^4(x-1)$ So $y' = 0$ when $x = 0, 1$.

when $x < 0$, $x^4$ is positive and $x - 1$ is negative; so $y'$ is negative and $y$ is decreasing.

when $0 < x < 1$, $x^4$ is pos and $x - 1$ is neg; so $y'$ is neg and $y$ is decreasing.

when $1 < x$, $x^4$ is pos and $x - 1$ is pos, so $y'$ is pos and $y$ is increasing.

$y'' = 30x^3(5x-4)$ So $y'' = 0$ when $x = 0, (4/5)$.

when $x < 0$, $x^3$ is neg and $5x - 4$ is neg; so $y''$ is pos and $y$ is concave-up.

when $0 < x < (4/5)$, $x^3$ is pos and $5x - 4$ is neg; so $y''$ is neg and $y$ is concave-down.

when $(4/5) < x$, $x^3$ is pos and $5x - 4$ is pos; so $y''$ is pos and $y$ is concave-up.

So the graph should be drawn so that
for $x < 0$ the graph is decreasing and concave-up;
for $0 < x < (4/5)$ the graph is decreasing and concave-down;
for $(4/5) < x < 1$ the graph is decreasing and concave-up;
for $1 < x$ the graph is increasing and concave-up.

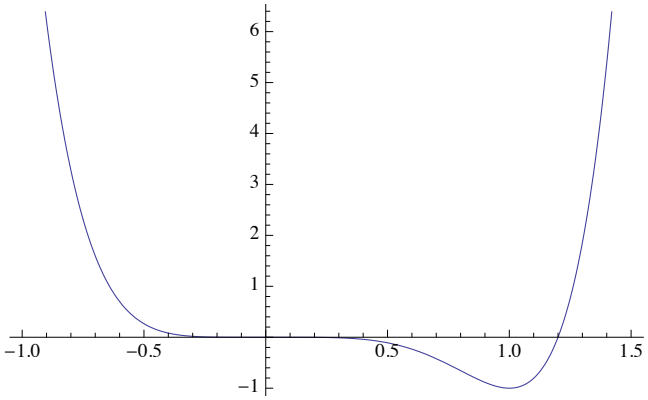


Figure 1: $y = 5x^6 - 6x^5$