Math 16a Problem List

-answers for selected problems-

Fall 2012

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(1) (a) does not exist (b) 1

(2) $(1, 1002)$ and $(-1, 1000)$

(3) $80(2003)^{15}$

(4) $y = (1/8)x + 6$

(5) $f'(x) = -6(2x+1)^2$ if $x \leq -1/2$ and $f'(x) = 6(2x+1)^2$ if $x \geq -1/2$

(6) -1

(7) $3/e$

(8) (a) $(0, 0)$ (b) $2/3$ (c) no, $g(x)$ is increasing on $(2, \infty)$.

(9) (a) $\frac{dy}{dx} = \frac{3x^2+y^2}{1-2xy}$ (b) $(1/\sqrt{2}, 1/\sqrt{2})$

(10) $30\sqrt{5}$

(11) $-1/3e$

(12) $(x^x)(1 + \ln x)$

(13) $y^{(2)} = e^x - (1/x^2)$; domain of $y^{(2)}$ is $(0, \infty)$.

(14) $1/2$

(15) -2

(16) (a) $(-\infty, 1) \cup (1, \infty)$ (b) $x > 1$ (c) no, $f(1)$ is not defined.

(17) $x \neq -2$, $x \neq 1$

(18) For constants C_1 and C_2,

$$F(x) = \begin{cases} \ln(x+1) + C_1 & : \quad x > -1 \\ \ln(-(x+1)) + C_2 & : \quad x < -1 \end{cases}$$

(19) $(e^{e^x + x})(e^x + 1)$

(20) (a) 1 (b) $1/2$

(21) $e^{(x-1)} + C$

(22) 1

(23) $2 - (1/80)$

(24) $32 - (1/16)$

(25) $(0, 0)$, $(0, \sqrt{3})$ and $(0, -\sqrt{3})$

(26) (a) $e^2 - e$ (b) $\pi/2$

(27) $e^{(x^2)}$

(28) $\left(\sqrt{3}/2, \ln\left[\frac{6\sqrt{3}+9}{16}\right]\right)$

(29) $6\frac{5}{6}$

(30)

(31) $(0, 0)$ if $b \leq 1/2$;
$\left([b - (1/2)]^{1/2}, b - (1/2)\right)$ and $\left(-[b - (1/2)]^{1/2}, b - (1/2)\right)$ if $b > 1/2$

(32) $4/15$

(33) a) $f'(x) = 2x - 3$ for $x < 1$ and for $x > 2$; $f'(x) = 3 - 2x$ from $1 < x < 2$; $f'(x)$ is not defined for $x = 1$ or for $x = 2$.
b) $f'(x) = -3(1+x)^2$ for $x \leq -1$, and $f'(x) = 3(1+x)^2$ for $x > -1$.

(34) $2e^{16}$

(35) (a) 0
(b) Concave up on the intervals $(-\infty, -1)$ and $(1, \infty)$
Concave down on the interval $(-1, 1)$.

(36) $(1/e)^{1/e}$

(37) (a) $y - b = \left(\frac{3a^2+1}{2b}\right)(x - a)$ (b) The tangent line is vertical

(38) 3

(39) $(-\frac{1}{2}, 2\frac{3}{4})$

(40) (a) no
(b) yes
(c) no

(41) (a)

$$f'(x) = \begin{cases} -3 & \text{if } x < -1 \\ -1 & \text{if } -1 < x < 0 \\ 1 & \text{if } 0 < x < 1 \\ 3 & \text{if } 1 < x \end{cases}$$

The domain of $f'(x)$ is $(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$.

(b)

$$f'(x) = \begin{cases} 1 & \text{if } x < 0 \\ 2x & \text{if } 0 < x < 1 \\ 3x^2 & \text{if } 1 < x \end{cases}$$

The domain of $f'(x)$ is $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$.

(c)

$$f'(x) = \begin{cases} 1 & \text{if } x < 0 \\ 3x^2 & \text{if } 0 < x \leq 2 \\ 6x & \text{if } 2 < x \end{cases}$$

The domain of $f'(x)$ is $(-\infty, 0) \cup (0, \infty)$.

(42) $y - (1/2)^{1/2} = x + (1/2)^{1/2}$ and $y + (1/2)^{1/2} = x - (1/2)^{1/2}$

(43) $(2\sqrt{2}, 2\sqrt{2})$ and $(-2\sqrt{2}, -2\sqrt{2})$

(44) Let $f(x) = x^{101} + x^{51} + x$. Thus

$$f'(x) = 101x^{100} + 51x^{50} + 1.$$

Therefore $f'(x) > 0$ for all x and so by the *First Derivative Rule*, $f(x)$ is increasing on $(-\infty, \infty)$. This implies that for each c, the equation

$$f(x) = c$$

can have at most one solution.

(45) See (44). The function $h(x) = xe^x$ is increasing on $(0, \infty)$ since $h'(x) > 0$ on $(0, \infty)$. $h(1) = e$ and so $h(1) > 1$. Since $h(x)$ is increasing on $(0, \infty)$, $h(x) > h(1)$ for all $x > 1$ and so for all x in $(1, 2)$, $xe^x > 1$.

(46) 5/6

(47) $-(1/x)^x(1 + \ln x)$

(48) Only one point because $(0, 1)$ is on the curve and curve is concave up everywhere.

(49)

(50) 20 for right endpoints, 8 for left endpoints

(51) e^{-a^2}

(52) 3

(53) $(\ln 5)5^x + (1/x)(1/\ln 7)$

(54) (a) $1/3 + 1/4 + 1/5 + 1/6 = 57/60 \sim .95$
(b) $1/2 + 1/3 + 1/4 + 1/5 = 77/60 \sim 1.28$
(c) $2/5 + 2/7 + 2/9 + 2/11 \sim 1.09$
$\ln 3 \sim 1.10$

(55) 2