Math 16a Problem List
Spring 2012

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1. For each of the following, determine if the limit exists and compute the limit if it does exist.
   (a) $\lim_{x\to 0}\left(x\sqrt{1+(1/x^2)}\right)$
   (b) $\lim_{x\to 0}\left(x^2\sqrt{1+(1/x^4)}\right)$
2. Find the points on the graph of $y = x^3 + 1001$ where the tangent is parallel to the line $y = 3x$.
3. Find the derivative of
$$y = (x^3 + x^2 + 2001)^{16}$$
at $x = 1$.
4. Find the equation of the line which is tangent to the curve
$$y = 4x^{1/4}$$
at the point where $x = 16$.
5. Suppose $f(x) = |2x + 1|^3$. Find $f'(x)$.

6. Suppose $f(x)$ is a function with domain $(0, \infty)$ and that

$$f(x) = \frac{x^2 - 4x + 3}{x^2 - 1}$$

for $x \neq 1$ and that $f(1) = a$. Suppose $f(x)$ is continuous at $x = 1$. Find a.

7. Using the derivative, find an approximate value of $\ln 3$ in terms of e by using the fact that $\ln e = 1$.

8. Suppose $g(x) = \frac{1}{3}x^3 - x$.

   (a) Identify the inflection points of $g(x)$ or explain why there are none.

   (b) Find the maximum value of $g(x)$ for $0 \leq x \leq 2$.

   (c) Does $g(x)$ have a mimimum value for $x > 2$? Why?

9. Consider the curve defined by the equation $x^3 + y^2x = y$.

   (a) Find dy/dx by implicit differentiation.

   (b) Find the point (a, b) on the curve where $a > 0$ and the line tangent to the curve is vertical.

10. Find the minimum possible value of $3a + 5b$ given that $a > 0$, $b > 0$ and that $a \cdot b = 75$.

11. Find the minimum value of $y = x^3 \ln x$ for $x > 0$.

12. Suppose $y = x^x$ for $x > 0$. Find y'.

13. Suppose $y = (\ln x) + e^x$. Find y''.

14. Suppose x and y are differentiable functions of t related for all t by the equation

$$y^2 - xy + x^2 = 1.$$

Suppose $x(16) = 0$ and $x'(16) = 1$. Find $y'(16)$.

15. Suppose $g(x) = |\ln x|$. Find $g'(1/2)$.

16. Suppose $f(x) = (x+1)/(x-1)$.

    (a) Where is $f(x)$ decreasing?

    (b) Where is $f(x)$ concave up?

    (c) Does $f(x)$ have an inflection point? Why?

17. Suppose $f(x) = |x^2 + x - 2|$. For which values of x does $f'(x)$ exist?

18. Find all the antiderivatives of $f(x) = 1/(x+1)$ on the domain of $f(x)$.

19. Find the derivative of $y = e^{(e^x+x)}$.

20. Compute the following limits.

    (a)
    $$\lim_{x\to 0} \frac{\ln(x+1)}{x}$$

    (b)
    $$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$$

21. Find
    $$\int \frac{e^{x-x^2}}{e^{1-x^2}} dx$$

22. Find the derivative of
    $$y = \ln(x^2 - 6x + 9)$$
    at $x = 5$.

23. Using the derivative, find an approximate value of $31^{1/5}$.

24. Suppose
    $$f(x) = x^4 + \frac{x^2 + x - 6}{x^2 - 4}$$
    for $x \neq 2$ and that the domain of $f(x)$ is $(0, \infty)$. Suppose that $f'(2)$ exists. Find $f'(2)$.

25. Find the points on the curve defined by $x^3 + y^3 = 3y$ where the line tangent to the curve has slope 0.

26. Compute the following definite integrals.

    (a) $\int_1^2 e^x dx$.

    (b) $\int_{-1}^1 \sqrt{1-x^2} dx$

27. Suppose for all x

$$f(x) = \int_0^x e^{t^2} dt.$$

    Find $f'(x)$

28. Find the points on the graph of $y = \ln(x^3 + x^4)$ where the tangent line is perpendicular to the line $y = 1 - x/4$.

29. Find $\int_0^2 |x^5 - x^3| dx$.

30. Compute the derivative of $f(x) = x^2 - x$ at $x = 1$ using the definition of the derivative as a limit.

31. Find all the points on the the graph of $y = x^2$ which are closest to the point $(0, b)$. Give your answer in terms of b.

32. Find the area of the region bounded by the curves $y = x^2$ and $y = x^4$.

33. Find the derivatives for each of the following functions.

    (a) $f(x) = |x^2 - 3x + 2|$.

    (b) $f(x) = |1 + x|^3$.

34. Suppose for all $x > 0$

$$g(x) = \int_0^{x^2+1} e^{(t^4)} dt.$$

    Find $g'(1)$

35. Let
$$f(x) = \frac{x^2+1}{x^2-1}$$
   (a) Find all the values of x at which $f(x)$ has a relative extreme point.
   (b) Find where $f(x)$ is concave up and where $f(x)$ is concave down.

36. Let $f(x) = x^x$ $(x > 0)$. Find the minimum value of $f(x)$.

37. Consider the curve defined by the equation
$$y^2 = x^3 + x$$
   (a) Suppose (a, b) is a point on the curve and (a, b) is not the point $(0, 0)$. Find the equation of the line tangent to the curve at (a, b).
   (b) What happens at the point $(0, 0)$?

38. Find the slope of the line tangent to the graph of
$$y = x^{(x^2+x+1)}$$
   at $x = 1$.

39. Find the point on the graph of $y = 2^{(x^2+x+1)}$ where the tangent line is perpendicular to the line
$$x = 0$$
   .

40. For each of the following functions determine if they are differentiable at $x = 0$.
   (a) $f(x) = \sqrt{|x|}$
   (b) $f(x) = x\sqrt{|x|}$
   (c) $f(x) = \sqrt{1+|x|}$

41. For each of the following functions, find its derivative and the domain of its derivative.

    (a) $f(x) = |x+1| + |x| + |x-1|$

    (b)
    $$f(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x \leq 1 \\ x^3 & \text{if } 1 < x \end{cases}$$

    (c)
    $$f(x) = \begin{cases} x & \text{if } x < 0 \\ x^3 & \text{if } 0 \leq x < 2 \\ 3x^2 - 4 & \text{if } 2 \leq x \end{cases}$$

42. Find the equations of the lines tangent to the curve
    $$x^2 + y^2 = 1$$
    which have slope 1.

43. Find the points on the hyperbola $xy = 8$ which is closest to the point $(0, 0)$.

44. Show that the equation $x^{101} + x^{51} + x = 2001$ has at most one solution.

45. Show that the equation $xe^x = 1$ has no solutions in the interval $(1, 2)$.

46. Suppose $f(x) = x^2 + x$, $0 \leq x \leq 1$. Find the area under the curve $y = f(x)$ from $x = 0$ to $x = 1$.

47. Suppose $f(x) = (1/x)^x$ for $x > 0$. Find $f'(x)$.

48. How many lines tangent to the curve, $y = e^{(x^2)}$, contain the point, $(0, 1)$? Why?

49. Show that the triangle formed by any line tangent to the hyperbola $xy = 1$ and the coordinate axes has area 2. (amazing fact).

50. Use a Riemann sum with $n = 3$ to estimate the area under the graph of $f(x) = x^2 + x$ from $x = 0$ to $x = 3$. Use the right endpoints of the subintervals.

51. Find the minimum value of $y = e^{-x^2}$ on $[-1/2, a]$ where $a > 1$.

52. Suppose $f(x)$ has domain $(-\infty, \infty)$ and that for all x, $f'(x) = 3f(x)$. Suppose that $f(1) = 1$. Find $f'(1)$.

53. Suppose that $g(x) = 5^x + \log_7(x)$. Find $g'(x)$.

54. Use a Riemann sum with $n = 4$ to estimate

$$\ln 3 = \int_1^3 (1/x)\ dx$$

(a) Use the right endpoints of the subintervals.

(b) Use the left endpoints of the subintervals.

(c) Use the midpoints of the subintervals.

55. Suppose $g(x)$ is an antiderivative of the function,

$$h(x) = 1/x^2.$$

Suppose that $g(1) = 1$ and $g(-1) = 1$. Find $g(-2) + g(2)$.