

MATH 16A, SUMMER 2008, FINAL EXAM

Name: \_\_\_\_\_

SID: \_\_\_\_\_

- Please draw a box around all of your final answers.
- You may *not* use a calculator on this exam.
- The exam consists of 13 questions. The point values for each question are indicated below, and also before the problem numbers.
- You will have 110 minutes to complete this exam. Please work carefully, and check your answers when you are done. Remember not to spend too much time on any one problem. If you get stuck on a difficult problem, move on to a problem that you know how to do, and come back to the difficult problem later.
- If you finish your exam early, you may turn it in, collect an exam solution, and leave the room.
- The exam will end at 10:00AM.
- Please do not write anything below this line.

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Problem 1 \_\_\_\_\_ (out of 6)

Problem 8 \_\_\_\_\_ (out of 6)

Problem 2 \_\_\_\_\_ (out of 6)

Problem 9 \_\_\_\_\_ (out of 5)

Problem 3 \_\_\_\_\_ (out of 18)

Problem 10 \_\_\_\_\_ (out of 6)

Problem 4 \_\_\_\_\_ (out of 6)

Problem 11 \_\_\_\_\_ (out of 6)

Problem 5 \_\_\_\_\_ (out of 12)

Problem 12 \_\_\_\_\_ (out of 5)

Problem 6 \_\_\_\_\_ (out of 12)

Problem 13 \_\_\_\_\_ (out of 6)

Problem 7 \_\_\_\_\_ (out of 6)

Total Score \_\_\_\_\_ (out of 100)

(1) (6 points) [Implicit Differentiation] Write the equation of the line that is tangent to the curve  $x^2 + y^2 = 4$  at the point  $(1, -\sqrt{3})$ .

(2) (6 points) [Related Rates] A 10-foot-long ladder is leaning against the side of a brick wall. If the top of the ladder is sliding down the wall vertically at a constant rate of 2 feet per second, how fast is the ladder sliding along the ground away from the wall when the top of the ladder is 6 feet from the ground?

(3) (18 points) [Differentiation] Compute the derivative of the following functions. Simplify your answer.

(a)  $y = x^2 + 2x + 75$

$$(b) f(t) = (2t^2 + 4)^6$$

$$(c) R = \frac{\ln d}{d^2}$$

$$(d) g(y) = (2y - 1)e^y$$

$$(e) r(w) = \frac{(3w-2)(4w+4)(w^2)(5w-9)}{2w+1}$$

(f)  $z(x) = x^x$

(4) (6 points) [Exponents and Logarithms] Solve for  $x$ .

(a)  $e^{2x} + 2e^x = 3$

(b)  $\ln(x + 1) - \ln(x - 2) = 1$ .

(5) (12 points) [Exponential Growth] A certain community of insects is growing at a rate that is 4 times the current insect population. Suppose that at time  $t = 0$  (where  $t$  is in weeks) there are 1000 insects.

(a) Find a formula for  $P(t)$ , the insect population at time  $t$ .

(b) What is the population at time  $t = 5$ ?

(c) How long will it take the insect population to triple?

(d) How many insects will there be when the population is increasing at a rate of 18000 insects per week?

(6) (12 points) [Compound Interest]

Suppose that you invest 400 in a savings account that earns 5% interest compounded continuously.

(a) Write a formula that gives the amount of money you can expect to have  $t$  years after making this deposit.

(b) How much money will you have after 2 years?

(c) How long do you have to wait until you have  $400e$  dollars?

(d) At what rate will your money be increasing when you have \$1000 in your account?

(7) (6 points) [Learning Curve] The percentage of calculus material from this course that you know is related to the amount of time that you study. Suppose that this percentage could be approximated by the learning curve function  $f(t) = 100(1 - e^{-.04t})$  where  $t$  is in hours of study time.

(a) If you wanted to learn 80% of the material, how long should you have studied?

(b) If you decided to keep studying until your rate of learning falls to 1% per hour, what percentage of the material would you have learned?

(8) (6 points) [Antidifferentiation]

(a) Compute  $\int (e^{3x} + \frac{3}{x^2} - \sqrt{x}) dx$

(b) Suppose  $f'(x) = e^{2x+1}$  and  $f(2) = 5$ . Find a formula for  $f(x)$ .

- (9) (5 points) [Fundamental Theorem of Calculus] Let  $F(x) = \int_6^x (t^2 + 7)dt$ , and let  $h(x) = 3x^2$ . Compute  $(F \circ h)'(1)$ .

- (10) (6 points) [The Definite Integral]  
Compute the following definite integrals.
- (a)  $\int_0^3 (e^{2x} + 1)dx$

(b)  $\int_1^4 (3x^2 + 2x + 1)dx$



(11) (6 points) [Areas between curves] Find the area between the curves  $y = x^2$  and  $y = -2 - x^2$  from  $x = -1$  to  $x = 1$ .

(12) (5 points) [Average Value of a Function] What is the average value of the function  $H(x) = e^x + 3x^2 + 1$  from  $x = 0$  to  $x = 2$ ?

(13) (6 points) [Volume] Find the volume of the solid that results from rotating the curve  $y = e^x$  about the  $x$ -axis from  $x = 0$  to  $x = 1$ .