Express each of the following linear equations in operator notation. In other words, write it as $L(y) = 0$ for some linear operator $L$. Factor the operators into a sequence of constant-coefficient operators:

1. $y'' - 2y' - 35y = 0$
2. $2y''' - 6y'' + 6y' - 2y = 0$

Rewrite each higher order equation as a system of first-order equations:

1. $y'' - 2y' + y = 0$
2. $y^{(3)} + 10y' + y = 0$

Rewrite each system of equations as a matrix equation (possibly reducing the order of the equations, as above, first):

1. $x' = y; y' = -x$
2. $x' = 2x + 2y + z; y' = x - y - z; z' = z - y$
3. $x' = e^t y; y' = t^2 x + ty$
4. $x'' + 2x' - 2y' + 3y = 0; y' = x'' - 2x$

Here are some morsels in the spirit of last week, with a view towards Wednesday’s quiz:

1. Solve the homogeneous equation $y^{(4)} - 2y^{(3)} + 3y'' - 4y' + 2y = 0$ (i.e., find the general solution)
2. Give an example of a nonlinear third-order differential equation.
3. Compute the Wronskian of $\sin 3x$ and $e^{2x}$.
4. Use variation of parameters to find a particular solution to $y'' + 2y' + y = e^{-t}$.
5. Find a particular solution to $y'' - 3y' - 4y = e^{3t}$, by any method you want.
6. (T/F) For any two functions $y_1$ and $y_2$, if the Wronskian $w[y_1, y_2]$ is never zero, then they are independent.
7. (T/F) If $y$ is a solution of the inhomogeneous equation $y'' + y' + y = 1$ satisfying the initial condition $y(2) = 1, y'(2) = 0$, then $y$ must be the constant function $y(t) = 1$ for all $t$.
8. (T/F) If the three functions $y_1, y_2$, and $y_3$ are linearly dependent, then the equation $c_1y_1(t) + c_2y_2(t) + c_3y_3(t) = 0$ holds for all values of $c_1, c_2$, and $c_3$.
9. (T/F) An $n$-th order homogeneous differential equation has a solution set which is an $n$-dimensional vector space.
10. Find all possible pairs $(a, b)$ such that the functions $\sin ax$ and $\cos bx$ are linearly independent. [Hint: For any values of $a$ and $b$, both functions are solutions to the equation $(D^2 + a^2)(D^2 + b^2)(y) = 0.$]