MATH 54 QUIZ 9

Solution

Write your name at the top. Questions (1) and (2) are worth one point each; (3) and (4) are worth four points each. You must show your work neatly and clearly and box your answers for full credit. There’s a question on the back of this page.

(1) (T/F) The function \( y = 2/x \) is a solution to the differential equation \( y'' + yy' = 0 \).

\[ \text{T} \]

(2) (T/F) If \( y_1 \) is a solution to \( x^2y'' - 4xy' + 6y = 0 \) and \( y_2 \) is a solution to \( x^2y'' - 4xy' + 6y = \cos x \), then \( y_1 + y_2 \) is a solution to \( x^2y'' - 4xy' + 6y = \cos x \).

\[ \text{T} \]

(3) (a) Use the method of undetermined coefficients to find a particular solution to the equation \( y'' - y = \sin 2x \).

\[ y_p = -\frac{1}{5} \sin 2x \]

Guess \( y_p = A \sin 2x + B \cos 2x \)

\[ y_p'' = y_p = -4A \sin 2x - 4B \cos 2x = 4A \sin 2x - 4B \cos 2x = -5A \sin 2x - 5B \cos 2x \]

\[ \Rightarrow A = -\frac{1}{5}, \quad B = 0 \]

\[ y_p = -\frac{1}{5} \sin 2x \]

In parts (b) and (c), give the general form used to find a particular solution for the equation — do NOT determine the coefficients.

(b) \( y'' + 2y' + y = (t + 1)e^{-t} \)

\( y_p = t^2 (Ae + B) e^{-t} \)

Since \(-1\) is a double root of aux eqn.

(c) \( y'' - y' - 12y = e^t \sin 3t + 4t^2 e^{2t} \)

\( e^t (A \sin 3t + B \cos 3t) + (Ct^2 + De + E) e^{2t} \)

1
(4) Use the method of variation of parameters to find the general solution to the equation $y'' - 2y' + y = e^{2t}$.

$$y = c_1 e^t + c_2 e^t + e^{2t}$$

$v_1 e^t + v_2 e^t = 0$

$v_1 e^t + v_2 (t+1) e^t = e^{2t}$

$v_2' e^t = e^{2t}$

$v_2' = v_2 = e^t$

$v_1' = -e^t (e^t - t e^t) = te^t$

$v_1 = -(t-1) e^t$

$y_p = v_1 y_1 + v_2 y_2 = -(t-1) e^t + e^t - te^t$

$y_p = e^{2t}$

$y = c_1 y_1 + c_2 y_2 + y_p$