MATH 54, QUIZ 8 (UNGRADED)

(1) Show that $e^x$ and $e^{2x}$ are linearly independent functions (in the vector space of all differentiable functions on $\mathbb{R}$). Do $1, x,$ and $x^2$ form a linearly independent set? Why or why not?

(2) Find all values of $a$ and $b$ for which the function

$$f(t) = e^{at} + b$$

is a solution to the differential equation

$$f''(t) + 2f'(t) + f(t) = 0.$$

(3) Find the general solution to the homogeneous linear differential equation

$$y'' + 4y' + 4y = 0.$$  

Then find a particular solution which satisfies the initial condition $y(0) = 1, y'(0) = 0.$