MATH 54 QUIZ 6

There are ten points total. Do your computations on the back and just write the answers on this front page. Put your name and section number above. All matrices are assumed to be square.

ACT ONE

(1) (2 points) Diagonalize the matrix \( A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} \). That is, find \( 2 \times 2 \) matrices \( P \) and \( D \) where \( P \) is invertible and \( D \) is diagonal, such that \( A = PDP^{-1} \). [You need not compute \( P^{-1} \)]

\[ P = \quad D = \]

(2) (1 point) Show that \( v = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) is an eigenvector of the matrix \( R = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \). Your answer should be a one-line computation.

(3) (1 point) Suppose that one of the eigenvalues of the matrix \( A \) is \(-1\). If \( v \) is an eigenvector of \( A \) associated to the eigenvalue \(-1\), what is \( A^{99}v \)?

(4) (2 points) Let \( A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \). For what values of \( a \) and \( b \) is \( A \) diagonalizable? [You must find all values of \( a, b \) for full credit]

ACT TWO - TRUE OR FALSE (ONE POINT EACH)

(1) The 0-eigenspace of a matrix \( A \) is equal to \( \text{Null}(A) \).
(2) The sum of two eigenvectors for \( A \) is also an eigenvector for \( A \).
(3) An \( n \times n \) matrix is diagonalizable if and only if the sum of the dimensions of its eigenspaces is \( n \).
(4) If \( \lambda_1 \) is an eigenvalue of \( A \) and \( \lambda_2 \) is an eigenvalue of \( B \), then \( \lambda_1 \lambda_2 \) is an eigenvalue of \( AB \).