Write your name at the top. Question (1) is worth two points, question (2) four points, and the remaining questions a point apiece. Please show your work neatly if you wish to be considered for partial credit.

(1) Express the third-order (scalar) differential equation \( y''' - 2y'' - y' + y = 0 \) as a first-order vector equation \( x' = Ax \). In other words, reduce it to a system of three first-order equations and express this system in matrix form.

\[
\begin{align*}
    x_1' &= y \\
    x_2' &= y' \\
    x_3' &= y''
\end{align*}
\]

\[ x' = A \dot{x} \quad \text{where} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix} \]

(2) Find the general solution to the vector differential equation \( x' = Ax \), where

\[
A = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 5 \\ 0 & 0 & -2 \end{bmatrix}
\]

\( A \) is diagonal, so its eigenvalues are the diagonal entries: \( \lambda_1 = 5, \lambda_2 = 0, \lambda_3 = -2 \).

\[
A - \lambda I = \begin{bmatrix} 0 & -8 & 1 \\ 0 & -5 & 5 \\ 0 & 0 & -7 \end{bmatrix} \sim \begin{bmatrix} 5 & -8 & 0 \\ 0 & -5 & 5 \\ 0 & 0 & -7 \end{bmatrix} \quad \Rightarrow \quad \text{e-vector} \quad v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
\]

\( \lambda_2 = 0 \)

\[
A - \lambda_2 I = \begin{bmatrix} 5 & -8 & 1 \\ 0 & 0 & 5 \\ 0 & 0 & -2 \end{bmatrix} \sim \begin{bmatrix} 5 & -8 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 5 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 8/5 \\ 0 \\ 1 \end{bmatrix} = v_2
\]

\( \lambda_3 = -2 \)

\[
A - \lambda_3 I = \begin{bmatrix} 7 & -8 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 7 & 0 & 2 \\ 0 & 2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 1/2 \\ 2/5 \\ 1 \end{bmatrix} = v_3
\]

General sol'n \( x = C_1 e^{5t} v_1 + C_2 e^{0t} v_2 + C_3 e^{-2t} v_3 \)