Write your name at the top. Question (1) is worth two points, question (2) four points, and the remaining questions a point apiece. Please show your work neatly if you wish to be considered for partial credit.

(1) Express the third-order (scalar) differential equation \( y''' - 2y'' - y' + y = 0 \) as a first-order vector equation \( \mathbf{x}' = A\mathbf{x} \). In other words, reduce it to a system of three first-order equations and express this system in matrix form.

(2) Find the general solution to the vector differential equation \( \mathbf{x}' = A\mathbf{x} \), where

\[
A = \begin{bmatrix}
5 & -8 & 1 \\
0 & 0 & 5 \\
0 & 0 & -2 \\
\end{bmatrix}
\]
(3) (T/F) If $X(t)$ is a matrix-valued function of $t$ which satisfies the matrix equation $X'(t) = AX(t)$, then all solutions of the vector equation $x' = Ax$ can be obtained as linear combinations of the columns of $X$.

(4) (T/F) If the matrix $A$ is

$$
A = \begin{bmatrix}
0 & 1 & 1 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix},
$$

then the matrix exponential $e^A$ is equal to

$$
\begin{bmatrix}
1 & 1 & 3/2 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{bmatrix}.
$$

(5) (T/F) If the matrix $A$ has complex eigenvalues $\lambda = 3i$ and $\overline{\lambda} = -3i$, with associated (complex) eigenvectors $\mathbf{v} = x + iy$ and $\overline{\mathbf{v}} = x - iy$, then a basis for the space of real solutions of the matrix equation $x' = Ax$ is given by

$$
\{ \cos 3t \mathbf{x} - \sin 3t \mathbf{y}, \cos 3t \mathbf{y} + \sin 3t \mathbf{x} \}.
$$

(6) (T/F) For any matrices $A$ and $B$, $e^{A+B} = e^A e^B$. 