MATH 54 QUIZ 10

Write your name at the top of the page.

(1) (1 point) If \( y \) is a solution to the differential equation \( y'' - 2y' + 2y = 1 \) which satisfies the initial condition \( y(0) = 1, y'(0) = 0 \), then \( y \) is the constant function \( y = 1 \).

(2) (3 points) Use the Wronskian to show that the two functions \( y_1 = e^{xt} \) and \( y_2 = e^{bt} \) are linearly independent if \( a \neq b \).

\[
W(t) = \begin{vmatrix}
    e^{at} & e^{bt} \\
    ae^{at} & be^{bt}
\end{vmatrix} = (b-a)e^{(a+b)t}
\]

This is non-zero if \( a \neq b \), hence \( \{e^{at}, e^{bt}\} \) are independent.

(3) (3 points) Find the general solution to the homogeneous fifth order equation \( y^{(5)} + 3y'' - 4y = 0 \).

\[
\begin{aligned}
\text{Auxiliary equation:} & \quad r^5 + 3r^3 - 4r = 0 \\
= r(r^4 + 3r^2 - 4) = 0 \\
r(r^2 + 4)(r^2 - 1) = 0 \\
r(r + 1)(r - 1)(r + 2i)(r - 2i) = 0
\end{aligned}
\]

roots: \( r = 0, -1, 1, \pm 2i \) \( \Rightarrow \ y = C_1 + C_2 e^{-t} + C_3 e^t + C_4 \sin(2t) + C_5 \cos(2t) \)

(4) (3 points) Find a particular solution to the inhomogeneous equation \( y'' - 3y' + 2y = e^{2t} \) using any method you like.

Try \( y_p = Ate^{2t} \) (since \( 2 \) is a single root of \( r^2 - 3r + 2 = (r-1)(r-2) = 0 \))

\[
\begin{aligned}
\text{Let} & \quad r = 0, \pm 1, 2 \quad \Rightarrow \quad y_p = A te^{2t} \\
y_p' & = A(2t+1)e^{2t} \\
y_p'' & = A(4t+4)e^{2t}
\end{aligned}
\]

Since \( y_p = te^{2t} \) works,

\[
\begin{aligned}
y'' - 3y' + 2y & = A[(4t+4) - 3(2t+1) + 2e^{2t}]e^{2t} \\
& = e^{2t} \\
\Rightarrow & \quad A = 1
\end{aligned}
\]