

**MATH 115, SUMMER 2012**  
**QUIZ 5**

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There are problems on both sides of this page, 20 points total. Write clearly and in complete sentences. If you need extra paper, ask me.

- (1) (2 points each) True or False. Explain your answer briefly.
- (a) There are infinitely many Pythagorean triples containing the integer 12.
  
  
  
  
  
  
  
  - (b) If a line  $L$  in  $\mathbb{R}^2$  whose slope is a rational number passes through a point  $(a, b)$ , and  $a$  and  $b$  are not rational numbers, then none of the points on  $L$  have rational number coordinates.
  
  
  
  
  
  
  
  - (c) If  $P$  is a property of a positive integer which is such that, whenever  $n$  has property  $P$ , then  $n - 2$  has property  $P$ , then there are no integers with property  $P$ .
  
  
  
  
  
  
  
  - (d) If an integer matrix  $A$  has nonzero determinant, then there is another integer matrix  $B$  such that  $AB = BA = I$ .
  
  
  
  
  
  
  
  - (e) If  $C$  is a curve defined by a degree 2 polynomial  $f(x, y)$ ,  $P$  is a rational point on  $C$ , and  $L$  is a line through  $P$  whose slope is a rational number, then  $L$  intersects  $C$  in another point, which is also a rational point.

- (2) (5 points) Let  $a$  and  $b$  be relatively prime integers. Explain how to choose  $c$  and  $d$  such that the matrix

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

is in the modular group.

- (3) (5 points) Explain why the equation  $7x^3 + 8y^2 = 818$  has no integer solutions.