

**MATH 115, SUMMER 2012**  
**QUIZ 4**

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There are two problems - one is on the back of this page, 20 points total. Write clearly and in complete sentences. If you need extra paper, ask me. You may find the following definition and theorems useful:

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**Definition 1.** A form  $f(x, y) = ax^2 + bxy + cy^2$  is **reduced** if either  $-a < b \leq a < c$  or  $0 \leq b \leq a = c$ .

**Theorem 1** (The Reduction Theorem). *Let  $f(x, y) = ax^2 + bxy + cy^2$  be a primitive positive definite QF (integral and binary as usual). Then*

- (1)  *$f$  is equivalent to a unique reduced form.*
- (2)  $|b| \leq a \leq \sqrt{-d/3}$

**Theorem 2.** *Let  $p$  be an odd prime and  $d$  any integer congruent to 0 or 1 mod 4. Then  $p$  is represented by a form of discriminant  $d$  if and only if  $d$  is a square mod  $p$ .*

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- (1) (10 points) Find a reduced form equivalent to  $37x^2 - 22xy + 13y^2$ .

- (2) (10 points) Which primes are represented by the form  $f(x, y) = x^2 + 3y^2$ ?  
Use Theorems 1 and 2 to carefully prove your answer.