MATH 115, SUMMER 2012
QUIZ 2

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There are problems on the back of this page. Write clearly and in complete sentences. If you need extra paper, ask me.

(1) (6 points) Solve the system of congruences, if possible. If not possible, explain why not.

$$\begin{aligned} x &\equiv 7 \mod 12 \\ x &\equiv 3 \mod 20 \end{aligned}$$

(2) (3 points) Explain why 101 divides $(100! + 1)$ [Hint: 101 is prime]

(3) (3 points) For which integers $m > 1$ does the ring $\mathbb{Z}/m$ have no zerodivisors?

(4) (3 points) The Chinese Remainder Theorem says that there is an isomorphism ψ from $\mathbb{Z}/4 \times \mathbb{Z}/5$ to $\mathbb{Z}/20$. What is $\psi(1, 3)$?

(5) (1 point each) True or False. No justification necessary.

(a) There are integers a, b such that $980 = a^2 + b^2$.

(b) If $f\colon R \to S$ is a ring homomorphism and $r \in R$ is a unit, then $f(r)$ is a unit in S.

(c) $6^{145} \equiv 1 \mod 13$.

(d) The congruence

$$8x \equiv 7 \mod 22.$$

has solutions.

(e) If $m > 1$ is odd, then $\phi(m) = \phi(2m)$ (here ϕ is Euler's totient function).