MATH 115, SUMMER 2012
QUIZ 1 SOLUTION
THURSDAY, JUNE 21ST

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Please show your work carefully and write neatly. Make sure to double-check your answers. There are 20 points total.

(1) (6 points) Calculate the greatest common divisor of 115 and 585. Then express this gcd as a linear combination of 115 and 585.

Solution:

$$585 = 5 \cdot 115 + 10$$
$$115 = 11 \cdot 10 + 5$$
$$10 = 2 \cdot 5 + 0$$

So 5 is the gcd. To write it as a linear combination, we have

$5 = 115 - 11 \cdot 10 = 115 - 11 \cdot (585 - 5 \cdot 115) = -11 \cdot 585 + 56 \cdot 115$

(2) (6 points) Explain why there are no integers x and y satisfying the equation

$$986x + 406y = -29$$

Solution: There is a solution to this equation if and only if -29 is a multiple of the gcd of 986 and 406, which we now compute:

$$986 = 2 \cdot 406 + 174$$
$$406 = 2 \cdot 174 + 58$$
$$174 = 3 \cdot 58 + 0$$

So the gcd is 58, which does not divide -29, so there's no solution. Alternatively, you could just observe that some other common divisor, not necessarily the greates, divides both 986 and 406, but not 29. For instance, 2.

(3) (True or False - 2 points each)

(a) If n is any integer, then n and $n^2 + n + 1$ are relatively prime.
True: Using tricks for gcd, $(n, n^2+n+1) = (n, n^2+n+1-(n+1)\cdot n) = (n, 1) = 1$.

(b) If a, b, and c are three integers, not all zero, and their gcd (a, b, c) is 1, then the gcd of ab and c is 1.

False: $a = c = 2$, $b = 3$ is a counterexample.

(c) If a, b, c are three integers such that $ac|bc$, then $a|b$.
True: By hypothesis, there exists k such that $bc = k \cdot ac$. c is nonzero, since if it were zero, we would be assuming that $0|bc$, but zero doesn't divide anything. Thus we may cancel the c and obtain $b = ka$, which means $a|b$.

(d) If a and b are two integers such that $(a, 4) = (b, 4) = 2$, then $(a+b, 4) = 4$.
True: Our assumptions on a and b mean that 2 divides both, but 4 divides neither. So each of a and b have the form $4k+2$, say $a = 4k_1+2$, and $b = 4k_2 + 2$. Thus $a + b = 4(k_1 + k_2) + 4$, which is divisible by 4. Thus the gcd is at least 4, but it can't be larger, since nothing larger than 4 divides 4.