MATH 115, SUMMER 2012
MOCK QUIZ, LECTURE 9

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Pretend this is a real quiz and take. It's not for a grade, though. First look for the ones you already know how to do. We'll go over it at the end, but use it as a guide to see what you should study most for tomorrow's (real) quiz.

(1) (CRT problem)

Solve the system of congruences, if possible. If not possible, explain why not.

$$x \equiv 13 \mod 20$$
$$x \equiv 3 \mod 15$$
$$x \equiv 9 \mod 12$$

(2) (Solving one linear congruence)

Find all integers x satisfying the congruence

$$12x \equiv 16 \mod 20.$$

Then give an integer b for which the congruence $12x \equiv b \mod 20$ has no solution, and say briefly why not.

(3) (Algebra Questions)

(a) Give an example of a ring, and an element of this ring which is neither a unit nor a zerodivisor.

(b) Explain why there are no ring homomorphisms from $\mathbb{Z}/2$ to $\mathbb{Z}/m$, for any $m > 2$.

(c) Compute $(2, 4) + (1, 3)$ and $(2, 4) \cdot (1, 3)$ in the ring $\mathbb{Z}/3 \times \mathbb{Z}/5$.

(d) The Chinese Remainder Theorem says that there is an isomorphism ϕ from $\mathbb{Z}/3 \times \mathbb{Z}/5$ to $\mathbb{Z}/15$. What is $\phi(2, 3)$?

(4) Suppose $p > 2$ is a prime that divides $26^2 + 64^2 = 4672$. What is the remainder when p is divided by 4?