MATH 115, SUMMER 2012
WS FOR LECTURE 7,8

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1. Computations

(1) Solve or say why there is no solution:
  (a) $16x \equiv 3 \mod 124$
  (b) $24x \equiv 4 \mod 23$

(2) Solve the system of congruences

$$\begin{aligned} x &\equiv 4 \mod 95 \\ x &\equiv 7 \mod 9 \\ x &\equiv 1 \mod 4 \end{aligned}$$

2. Ring Problems

(1) Is the set

$$\left\{ \begin{pmatrix} a & b \\ 4b & a \end{pmatrix} \middle| a, b \in \mathbb{Z} \right\}$$

a ring?

(2) Recall that a **unit** in a ring R is an element r for which there is another element r' such that $rr' = r'r = 1$. Prove that if $f \colon R \to S$ is a ring homomorphism and $r \in R$ is a unit, then $f(r)$ is a unit in S. Use this to give another proof that $\phi(mn) = \phi(m)\phi(n)$ if $(m, n) = 1$. [Note: you need the notion of an **isomorphism** to explain this succinctly, which we didn't get to in lecture]

(3) Suppose m, n are two integers greater than 1 and that there is a ring homomorphism $f \colon \mathbb{Z}/m \to \mathbb{Z}/n$. What can be said about the relationship between m and n?

3. Harder Problems

(1) (NZM 2.3.34) Prove that there is no positive integer n for which $\phi(n) = 14$. [Hint: First prove the following fact: if $p|n$, then $\phi(p)|\phi(n)$]

(2) (NZM 2.3.42) Find all positive integers n for which $\phi(n)|n$.