MATH 115, SUMMER 2012
WS FOR LECTURE 5,6

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(1) Find all solutions to the following congruences:
   (a) $2x \equiv 1 \mod 3$
   (b) $9x + 23 \equiv 28 \mod 25$
(2) Prove that $23 | a^{154} - 1$ whenever $(a, 23) = 1$.
(3) (slightly harder) If p is a prime such that $\frac{p-1}{2} \equiv 3 \mod 4$, show that $1 \cdot 2 \cdots (\frac{p-1}{2}) \equiv \pm 1 \mod p$. [Possible hint: use tricks similar to those in the proof of the $x^2 \equiv -1$ Thm]
(4) Prove that $\frac{1}{5}n^5 + \frac{1}{3}n^3 + \frac{7}{5}$ is an integer, for all $n \in \mathbb{Z}$.
(5) Prove that $n^{13} - n$ is divisible by 5 for any n. Is it divisible by any other numbers for all n?
(6) (Harder) Let p be prime. Show that $a^p \equiv b^p \mod p$ implies $a^p \equiv b^p \mod p^2$.