

MATH 115, SUMMER 2012
WEDNESDAY, JUNE 20TH
WORKSHEET

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- (1) Prove the theorem that relates gcd and lcm, namely: if a and b are not both zero, then $(a, b) \cdot [a, b] = |ab|$.
- (2) True or False? Give reasons for the true and provide counterexamples for the false:
 - (a) If p is prime and $p|a^3$, then $p|a$.
 - (b) If p is prime, such that $p|a$ and $p|(a^2 + b^2)$, then $p|b$.
 - (c) If $b|(a^2 + 1)$ then $b|(a^4 + 1)$.
 - (d) $(a, b, c) = ((a, b), (a, c))$

Divisibility Tests¹:

- (3) Prove that an integer is divisible by 3 if and only if the sum of its digits is divisible by 3.
- (4) Same as above, with 9 replacing 3.
- (5) Prove that an integer is divisible by 11 if and only if the difference between the sum of its even digits and the sum of its odd digits is divisible by 11.
Example: 30245 is divisible by 11 if and only if $(3 + 2 + 5) - (0 + 4) = 6$ is divisible by 11, so it ain't.
- (6) Show that if n is composite and $n > 4$, then $n|(n - 1)!$.
- (7) (Harder) Show that if a and b are relatively prime, then $(a + b, a^2 - ab + b^2)$ must be either 1 or 3.

¹These are easier to prove using modular arithmetic, but even if you've seen that before, try to prove knowing just what we know from this course