MATH 115, SUMMER 2012
PREPARATION FOR QUIZ 5

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1. What to Know

Be sure you know:

(1) What the modular group is.
(2) When an integer matrix is invertible (i.e., there exists an inverse matrix whose entries are integers).
(3) The classification of Pythagorean triples.
(4) The basic idea of the "geometric solution" of the Pythagorean Triples theorem.
(5) How the method of descent works.
(6) What a rational point on a curve is.
(7) How to tell that an equation has no integer or rational solutions by looking at the equation mod n for an appropriate choice of n.

2. Examples

(1) Is
$$\begin{pmatrix} 2 & -3 \\ 4 & -7 \end{pmatrix}$$
an invertible integer matrix?

(2) Does the equation
$$12x^2 + 13y^3 = 378$$
have a solution in integers? Why or why not?

(3) (T/F) If x, y, z is a primitive Pythagorean triple, then one of them is congruent to -1 mod 4, one of them is congruent to 0 mod 4, and one of them is congruent to 1 mod 4.

(4) (T/F) Every degree 2 curve contains rational points.

(5) Prove, using a result from class if necessary, that the area of a right triangle whose non-hypotenuse side lengths are integers cannot be a perfect square integer.