

1) Did in class yesterday. The answer was $x^2 + y^2$, and the steps used in the algorithm were: ①, ② w/ $k=4$, ①, ② w/ $k=-3$, ①, ② w/ $k=1$.

2) If we had a representation that was not proper, we would have $p = ax^2 + bxy + cy^2$, w/ $(x,y) \in g^2$, so $g^2 | p$. This is impossible.

3) For f to be pos def, we must have $f(x,y) = 0 \Rightarrow x=y=0$. But for $f(x,y) = x^2$, we have $f(0,1) = 0$ even though $(0,0) \neq (0,1)$.

4) Use the identity $(x_1^2 + D_{Y_1}^2)(x_2^2 + D_{Y_2}^2) = (x_1 x_2 + D_{Y_1 Y_2})^2 + D(x_1 y_2 - x_2 y_1)^2$

5) Write $f(x,y) = f(\vec{x}) = \vec{x}^T M \vec{x}$, w/ M the 2×2 symmetric matrix. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Then $f(\vec{x}) = f(A\vec{x}) \quad \forall \vec{x}$ implies that

$$\begin{aligned} \vec{x}^T M \vec{x} &= (A\vec{x})^T M (A\vec{x}) \\ &= \vec{x}^T (A^T M A) \vec{x} \end{aligned}$$

$$\text{so } M = A^T M A, \text{ hence } \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = A^T \begin{pmatrix} * & 0 \\ 0 & -1 \end{pmatrix} A$$

$$\text{Let } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ then } A^T \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} A = A^T \begin{pmatrix} a & -b \\ c & -d \end{pmatrix} A$$

$$\begin{aligned} \text{Setting this } &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ means } \begin{aligned} a^2 + c^2 &= 1 \\ b^2 + d^2 &= 1 \\ ab = -cd & \end{aligned} \\ &= \begin{pmatrix} a^2 + c^2 & -ab - cd \\ ab + cd & -b^2 - d^2 \end{pmatrix} \end{aligned}$$

This gives $A = \begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}$, four choices

Worksheet

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