

MATH 115, SUMMER 2012
WORKSHEET FOR LECTURE 21

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- (1) Using the algorithm given in the example from the end of lecture, find a reduced form equivalent to $f(x, y) = 458x^2 + 214xy + 25y^2$.
- (2) Prove that if a quadratic form $f(x, y)$ represents a prime p , then it represents p *properly*.
- (3) Let $f(x, y) = x^2$. What's wrong with the following argument? Since $x^2 \geq 0$ for all x , and $x^2 = 0$ if and only if $x = 0$, f is a positive definite form.
- (4) Let $f_D = x^2 + Dy^2$. Prove that if m and n are represented by f_D , then so is mn . Thus for these quadratic forms, at least, it suffices to determine which primes they represent.
- (5) Let $f(x, y) = x^2 - y^2$. Find all integer matrices A such that

$$f(\mathbf{x}) = f(A\mathbf{x})$$

for all $(x, y) \in \mathbb{Z}^2$. Matrices with this property are called **automorphs** of f .