MATH 115, SUMMER 2012
TUESDAY, JUNE 19TH
WORKSHEET

JAMES MCIVOR

(1) Which of the following subsets of $\mathbb{Z}$ are ideals? Why or why not?
   (a) The set of all prime numbers.
   (b) The set of all integer solutions x to the equation $2x + 12 = 0$
   (c) The set of all integers y such that there is an integer x with $y = 4x - 2$.
   (d) The set of integers of the form $3k + 1$.

(2) Let I_1 be the ideal generated by 5, and I_2 the ideal generated by 6. Describe the set of integers which are in both I_1 and I_2. Is it an ideal?

(3) (quiz 1 question!) Find the gcd of 542 and 78, and express it as a linear combination of these two numbers. No calculators allowed!

(4) (quiz 1 question!) Explain why there are no integers x and y such that

$$112x + 320y = 24$$

(5) (quiz 1 question!) Find integers x and y such that

$$318x - 96y = -12$$

(6) (NZM 1.2.7) Exhibit three integers that are relatively prime but not relatively prime in pairs.

(7) (NZM 1.2.14) Prove that if n is odd, $n^2 - 1$ is divisible by 8.

(8) (NZM 1.2.34) Prove that for all integers a, k not both zero, $(a, a + k)|k$. Write one proof using the language of ideals, and one without (they should be very similar)

(9) (Harder - NZM 1.2.50) Show that if a and b are relatively prime, then $(a + b, a^2 - ab + b^2)$ must be either 1 or 3.