MATH 110 WORKSHEET, JUNE 25TH

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Complex numbers

Let $z_1 = -i$, $z_2 = -1 + 2i$, and $z_3 = -\sqrt{2} - \sqrt{2}i$.

(1) Draw z_1, z_2, z_3 in the complex plane.

(2) Compute $z_2 + z_3$, $z_1 z_2$, and $z_2 z_3$.

(3) Find $|z_3|$, z_1^{-1}, and z_2^{-1}.

(4) Write z_1, z_2, and z_3 in polar form. Use this to compute $z_2 z_3$ and check you get the same thing as above.

(5) (bonus) What is $\sqrt{i}$?

Vector Spaces

(1) Which of the following are vector spaces over $\mathbb{R}$? If they aren't, which axioms don't hold?

(a) The set of all real polynomials which have no odd powers of x in them [example: $x^4 + 3x^2 + 2$ is OK, but $x^4 + 3x^2 + 5x + 2$ is not].

(b) The set of all vectors in $\mathbb{R}^3$ whose entries add up to 3.

(c) The set of all vectors in $\mathbb{R}^3$ whose entries add up to zero.

(2) We've seen that $\mathbb{R}^2$ is a real vector space. But we can try to make $\mathbb{R}^2$ into a *complex* vector space as follows: we already know how to add two vectors - the usual way for vectors in $\mathbb{R}^2$. To try to define scalar multiplication, we pick any complex number $a + bi$, and any vector $\begin{pmatrix} x \\ y \end{pmatrix}$, and set

$$(a + bi)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax - by \\ ay + bx \end{pmatrix}$$

Do you think that with these definitions of addition and scalar multiplication, we satisfy all the axioms of a vector space over $\mathbb{C}$?

(3) How many vectors are in the vector space $\mathbb{F}_5^2$? How about $\mathbb{F}_3^3$?