MATH 110 WORKSHEET, JULY 3RD

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(1) (Axler 2.10) If V is an n-dimensional space, prove that there exist one-dimensional subspaces $U_1, \ldots, U_n$ such that

$$V = U_1 \oplus \cdots \oplus U_n$$

(2) True or false: If $V = U \oplus W$, and (v_1, v_2, v_3) is a basis for V, and (v_1, v_2) is a basis for U, then (v_3) must be a basis for W.

(3) Consider $V = \mathbb{C}^2$ as a real vector space. We saw yesterday that it is 4-dimensional as a real vector space. Let

$$U = \left\{ \begin{pmatrix} a \\ bi \end{pmatrix} \middle| a, b \in \mathbb{R} \right\}$$

- (a) Prove that U is a subspace of V.
- (b) Find the dimension of U by finding a basis for U.
- (c) If we consider $\mathbb{C}^2$ as a 2-dimensional complex vector space instead (i.e., now we allow complex numbers for scaling), is the set U a subspace? If so, what is its dimension?