MATH 110 WORKSHEET, JULY 9TH

JAMES MCIVOR

(1) Consider the operator $T\colon P(\mathbb{F}) \to P_(\mathbb{F})$ given by $Tp(x) = xp'(x)$.
  (a) Is T an isomorphism?
  (b) Let n be any natural number. Prove that $P_n(\mathbb{F})$ is an invariant subspace for T (you don't need to check it's a subspace - just that it's invariant).

(2) Which of the following are invariant subspaces for the operator $T\colon \mathbb{F}^\infty \to \mathbb{F}^\infty$ given by $T(a_1, a_2, a_3, \ldots) = (a_2, a_3, a_4, \ldots)$?
  (a) $W = \{(a, a, a, \ldots) \mid a \in \mathbb{F}\}$
  (b) $U = \{(a_1, a_2, a_3, \ldots) \mid a_{2k} = a_{2k-1} \text{ for all } k \in \mathbb{N}\}$

(3) Consider the linear operator $T\colon P_2(\mathbb{F}) \to P_2(\mathbb{F})$ given by $Tp(x) = x^2 p(1)$.
  (a) Find all eigenvalues and eigenvectors for T.
  (b) Compute the matrix of T with respect to the basis $(1, 1-x, 1-x^2)$.

(4) Let T be an operator on V. Prove that the set of eigenvectors with eigenvalue zero is equal to the null space of T.