MATH 110 WORKSHEET, AUGUST 5TH

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(1) Consider the functional ϕ on $P_2(\mathbb{R})$ which takes a polynomial $p(x)$ to the real number $\int_0^1 p(x)\,dx$. Find a "representing vector" for this functional, i.e., a vector $q(x)$ such that $\phi(p(x)) = \langle p(x), q(x)\rangle$ for all $p(x)$. Use the inner product $\langle p(x), q(x)\rangle = \int_0^1 p(x)q(x)\,dx$.

(2) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ be the map $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+y \\ y+2z \\ z+x \\ y \end{pmatrix}$. Find a formula for T^*. Both spaces are given the standard (Euclidean) inner product.

(3) Let $T \in \mathcal{L}(V)$. Prove that $\operatorname{Null} T^* = (\operatorname{Range} T)^\perp$. (it's in the book if you get stuck...)

(4) Let $R\colon V \to V$ be a linear map such that $R^2 = I$ (R is a "reflection").
   (a) Show that you can find subspaces U and W of V such that $V = U \oplus W$ and $R\big|_U = I$, $R\big|_W = -I$.
   (b) Show that U is orthogonal to W if and only if $R = R^*$.