Math 110, Summer 2013
Instructor: James McIvor
Homework 3
Due Wednesday, July 17th
+2 bonus points for submitting it on Monday, July 15th

(1) (Axler 3.10) Prove that there does not exist a linear map $\mathbb{F}^5 \to \mathbb{F}^2$ whose null space is $\{(x_1, \ldots, x_5) \mid x_1 = 3x_2, x_3 = x_4 = x_5\}$.

(2) (Axler 3.23) Suppose that V is finite-dimensional and $S, T \in \mathcal{L}(V)$. Prove that $ST = I$ if and only if $TS = I$. Here ST and TS are shorthand for $S \circ T$ and $T \circ S$. (this is useful - it means when you're checking that two maps are inverses, you only need to check one of the two equations)

(3) Let $T\colon V \to W$ and $S\colon W \to U$ be linear maps. Prove that ST is the zero map if and only if $\operatorname{Range} T \subseteq \operatorname{Null} S$ (recall that ST is the zero map means $(ST)v = 0$ for all v in V).

(4) Find a linear map $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ whose null space is $U = \{(x, y, z, w) \in \mathbb{R}^4 \mid x = w, 2y = z\}$ and whose range is $W = \{(x, y, z) \in \mathbb{R}^3 \mid y = z\}$.

(5) (a) Prove that the map $T\colon P_2(\mathbb{F}) \to P_3(\mathbb{F})$ given by $Tp(x) = p'(x) - xp(x)$ is injective.
(b) Prove that for every $c \in \mathbb{F}$ (including $c = 0$), the evaluation map $T_c\colon P(\mathbb{F}) \to \mathbb{F}$ as defined in the previous HW is surjective.

(6) Let T be the map of problem 5(a). Using the bases $B_1 = (x^2, x, 1)$ and $B_2 = (1, 1 - x^2, x, x^3)$ for $P_2(\mathbb{F})$ and $P_3(\mathbb{F})$, respectively, compute $M(T, B_1, B_2)$.

(7) If $T\colon V \to V$ is a linear map whose matrix with respect to the basis $(v_1, \ldots, v_5)$ is

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{pmatrix},$$

find Tv, where $v = v_1 + v_2 + v_3 + v_4 + v_5$.