Math 110, Summer 2013
Instructor: James McIvor
Homework 1
Due Wednesday, July 3rd

(1) (a) Write $\frac{1+i}{1-i}$ in the form $a+bi$, for some $a, b \in \mathbb{R}$.
(b) Find all complex numbers z which satisfy $z^2 = -4i$.

(2) Axler, Chapter 1 problem 3: Prove that for every vector v in V, $-(-v) = v$ (in other words, prove that v is the additive inverse of $-v$.)

(3) Axler, Chapter 1 problem 4: Prove that if $a \in \mathbb{F}$, $v \in V$ and $av = 0$, then either $a = 0$ or $v = 0$.

(4) Axler, Chapter 1 problem 8: Prove that the intersection of any collection of subspaces of V is itself a subspace of V.

(5) Prove that $\{p(x) \in P(\mathbb{F}) \,|\, p'(x) = 0\}$ is a subspace of $P(\mathbb{F})$.

(6) Let $V = \mathbb{R}^3$, and let $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \,\middle|\, x + z = 0 \right\}$.
(a) Find a subspace W_1 of $\mathbb{R}^3$ such that $V \neq U + W_1$.
(b) Find a subspace W_2 of $\mathbb{R}^3$ such that $V = U + W_2$ but $V \neq U \oplus W_2$.

(7) Let $V = P_2(\mathbb{F})$, the space of polynomials of degree at most two, with coefficients in $\mathbb{F}$.
(a) Find examples of subspaces U and W of V such that $V \neq U + W$
(b) Find examples of subspaces U and W of V such that $V = U + W$ but $V \neq U \oplus W$.

(8) Find a polynomial $p(x)$ such that $(1 + x + x^2, 1 - x + x^2, p(x))$ spans $P_2(\mathbb{F})$.

(9) Consider the subspace $W = \{(x, y, z, w) \in \mathbb{R}^4 \,|\, 2x = z, y = 2w\}$ of $\mathbb{R}^4$.
(a) Find a list of vectors in W which spans W but is not linearly independent.
(b) Find a list of vectors in W which is linearly independent but does not span W.
(c) Find a basis for W.

(10) Axler, Chapter 2 problem 2: Prove that if $(v_1, \ldots, v_n)$ is linearly independent in V, then so is the list $(v_1 - v_2, v_2 - v_3, \ldots, v_{n-1} - v_n, v_n)$.

(11) Axler Chapter 2 problem 3: Suppose $(v_1, \ldots, v_n)$ is a linearly independent list in V and w is some vector in V. Prove that if the list $(v_1 + w, v_2 + w, \ldots, v_n + w)$ is linearly dependent, then w must be in the span of $(v_1, \ldots, v_n)$.

(12) Let E be the subset of $P_5(\mathbb{F})$ consisting of *even* polynomials (this means they must satisfy $p(-x) = p(x)$). Prove that E is actually a subspace of $P_5(\mathbb{F})$, find a basis for E, and prove that your answer is actually a basis.