(1) (a) Write $\frac{1+i}{1-i}$ in the form $a+bi$, for some $a, b \in \mathbb{R}$.
   (b) Find all complex numbers $z$ which satisfy $z^2 = -4i$.

(2) Axler, Chapter 1 problem 3: Prove that for every vector $v$ in $V$, $-(-v) = v$ (in other words, prove that $v$ is the additive inverse of $-v$).

(3) Axler, Chapter 1 problem 4: Prove that if $a \in \mathbb{F}$, $v \in V$ and $av = 0$, then either $a = 0$ or $v = 0$.

(4) Axler, Chapter 1 problem 8: Prove that the intersection of any collection of subspaces of $V$ is itself a subspace of $V$.

(5) Prove that $\{p(x) \in P(\mathbb{F}) \mid p'(x) = 0\}$ is a subspace of $P(\mathbb{F})$.

(6) Let $V = \mathbb{R}^3$, and let $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + z = 0 \right\}$.
   (a) Find a subspace $W_1$ of $\mathbb{R}^3$ such that $V \neq U + W_1$.
   (b) Find a subspace $W_2$ of $\mathbb{R}^3$ such that $V = U + W_2$ but $V \neq U \oplus W_2$.

(7) Let $V = P_2(\mathbb{F})$, the space of polynomials of degree at most two, with coefficients in $\mathbb{F}$.
   (a) Find examples of subspaces $U$ and $W$ of $V$ such that $V \neq U + W$.
   (b) Find examples of subspaces $U$ and $W$ of $V$ such that $V = U + W$ but $V \neq U \oplus W$.

(8) Find a polynomial $p(x)$ such that $(1 + x + x^2, 1 - x + x^2, p(x))$ spans $P_2(\mathbb{F})$.

(9) Consider the subspace $W = \{(x, y, z, w) \in \mathbb{R}^4 \mid 2x = z, y = 2w\}$ of $\mathbb{R}^4$.
   (a) Find a list of vectors in $W$ which spans $W$ but is not linearly independent.
   (b) Find a list of vectors in $W$ which is linearly independent but does not span $W$.
   (c) Find a basis for $W$.

(10) Axler, Chapter 2 problem 2: Prove that if $(v_1, \ldots, v_n)$ is linearly independent in $V$, then so is the list $(v_1 - v_2, v_2 - v_3, \ldots, v_{n-1} - v_n, v_n)$.

(11) Axler Chapter 2 problem 3: Suppose $(v_1, \ldots, v_n)$ is a linearly independent list in $V$ and $w$ is some vector in $V$. Prove that if the list $(v_1 + w, v_2 + w, \ldots, v_n + w)$ is linearly dependent, then $w$ must be in the span of $(v_1, \ldots, v_n)$.

(12) Let $E$ be the subset of $P_5(\mathbb{F})$ consisting of even polynomials (this means they must satisfy $p(-x) = p(x)$). Prove that $E$ is actually a subspace of $P_5(\mathbb{F})$, find a basis for $E$, and prove that your answer is actually a basis.