Math 202B— UCB, Spring 2014 — M. Christ
Problem Set 6, due Wednesday March 3

(6.1) Folland 5.48. □

(6.2) Folland 5.49. □

(6.3) (Folland 5.51) Let X be a normed vector space and $V \subset X$ a subspace. Show that V is norm-closed if and only if V is weakly closed. □

(6.4) Folland 5.52. □

(6.5) (Folland 5.56) Let V be a subspace of a Hilbert space $\mathcal{H}$. Then $(V^{\perp})^{\perp}$ equals the smallest closed subspace of $\mathcal{H}$ that contains V. □

(6.6) Folland problem 5.57. □

(6.7) Folland problem 5.58. □

(6.8) (Folland 5.59) Let K be a nonempty closed convex subset of a Hilbert space $\mathcal{H}$. Then K contains a unique element with smallest norm. □

Hints

□

(6.8) If $\mathcal{M}$ is a closed subspace of $\mathcal{H}$ and $x \in \mathcal{H}$, then $K = \mathcal{M} - x$ is a closed convex set. Therefore a corollary of this problem is the fact that $\mathcal{M}$ contains a unique element closest to x. Therefore you should probably either use this corollary, or employ a method that is sufficiently powerful to prove the corollary. □