Approximations and Errors – An Example

- Consider $\boxed{\int_1^2 \frac{1}{x}\, dx}$.
  The exact value of this definite integral is $\ln(2) \approx 0.69314718\ldots$.
- The table shows approximations to this exact value obtained from the left endpoint (L_n) and midpoint (M_n) methods in columns 2 and 3.
- In columns 4 and 5 it shows the errors – the differences between these approximations and the exact value.

n	L_n	M_n	E_{L_n}	E_{M_n}
5	0.745635	0.691908	-0.052488	0.001239
10	0.718771	0.692835	-0.025624	0.000312
20	0.705803	0.693069	-0.012656	0.000078

Some Points To Notice

- As n increases, both approximations L_n and M_n get better.
- For $n = 20$, L_n is accurate to the 2nd decimal place. Not bad, but M_n is accurate to the 4th!
- Question: Which requires more work to calculate, L_n or M_n?
- Answer: They require about the same amount of work.

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Some Points To Notice

- Question: Which method would a sensible person choose, left endpoint or modpoint?
- Answer: The midpoint method.

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Questions

- The midpoint method seems to be better.
- How much better? Does this example illustrate a general principle? Or is it just a fluke?
- Suppose someone asked us to calculate the answer to 10 decimal places. How large an n would we need to use?

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