Mathematics 258 – Fall 2016
Introduction to Euclidean Harmonic Analysis

Instructor: Michael Christ
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Office hours: Tuesday 10-11, Friday 2:15-3:15.
Course meets: MWF 11:10-12:00 in 2 Evans Hall
Text: Lecture notes *Euclidean Harmonic Analysis* by M. Christ. (These notes will be distributed in installments throughout the semester at no charge.)
bCourses: This course will have a bCourses site, on which lecture notes will be available.
Prerequisite: Math 202AB (graduate real/functional analysis), or equivalent with permission of instructor.

This is an introductory course at the second year graduate level. It will treat harmonic analysis in Euclidean spaces and allied topics in real analysis. More details will be glossed over in lectures than in a typical first year course.

Topics: Most of the following, insofar as time permits.

- Fourier transform and series. Functorial properties, inversion, Poisson summation, localization, symmetry, identities. Fourier transform on finite cyclic groups.
- Schwartz space, tempered distributions, approximations to the identity.
- Classical theory of convergence and divergence of Fourier series and integrals in dimension 1. Uniform, L^p, and almost everywhere convergence. Pointwise divergence. Maximal operators. Decay of Fourier coefficients.
- Convolution. Inequalities. Algebras of L^1 functions and finite measures. Wiener's theorem.
- Connections with complex analysis and harmonic functions. Boundary values, Dirichlet problem, conjugate function. F. and M. Riesz theorem.
- Hardy-Littlewood maximal function, stopping time constructions, $L^{1,\infty}$, John-Nirenberg inequality, good λ inequalities.
- The Calderón-Zygmund method and singular integral operators.
- Interpolation of operators. The real and complex methods. Lorentz spaces. Analytic families of operators.
- Almost orthogonality. Littlewood-Paley theory and Carleson measures.
- Fourier multiplier operators.
- Oscillatory integrals. Stationary phase, van der Corput's lemma, connections with curvature.
- Convergence of multiple Fourier series. Introduction to the Kakeya problem, Bochner-Riesz multipliers, Fourier restriction inequalities, and Strichartz inequalities.
- $T(1)$ Theorem and paraproducts.
- Sharp forms and extremizers of certain classical inequalities.

Required work: Either (a) solve and submit a reasonable number of exercises from the text, or (b) write a report on a topic relevant to the course. Typically this will involve reading one or several research or expository articles, and writing a summary (roughly 10 pages in length). A list of suggested topics and references will be distributed. Students are encouraged to find their own topics, subject to instructor's approval. Undergraduates are expected to complete option (a). There will be no exams.

References

The following books are relevant to the course and (together) contain a great deal of supplemental or more advanced material.

- R. R. Coifman and Y. Meyer, *Ondelettes et Opérateurs*, Vol. 3, Hermann, 1991.
- L. Grafakos, *Classical Fourier Analysis*, Springer
- L. Grafakos, *Modern Fourier Analysis*, Springer
- I. Katznelson, *An Introduction to Harmonic Analysis*, Dover, New York, 1976.
- T. W. Körner, *Fourier Analysis*, Cambridge University Press, 1988.
- Y. Meyer, *Wavelets and operators*, Cambridge University Press, 1992.
- Y. Meyer, *Ondelettes et Opérateurs*, Vols. 1,2, Hermann, 1990.
- C. Muscalu and W. Schlag, *Classical and Multilinear Harmonic Analysis*, 2 volumes, Cambridge University Press.
- E. M. Stein and R. Shakarchi, *Fourier Analysis*, Princeton University Press.
- E. M. Stein, *Singular Integrals and Differentiability Properties of Functions*, Princeton Univ. Press, Princeton, 1970.
- E. M. Stein, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton Univ. Press, Princeton, 1993.
- E. M. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton Univ. Press, Princeton, 1971.
- T. H. Wolff, *Lectures on Harmonic Analysis*, American Mathematical Society, 2003.
- A. Zygmund, *Trigonometric Series*, Cambridge Univ. Press, 1959.